What Is the Einstein Field Equation, and Why Does It Matter for Quantum Gravity?

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Gefördert durch die Deutsche Forschungsgemeinschaft (DFG) – Projektnummer 312032894, CU 338/1-2.
(Funded by the German Research Foundation (DFG) – project number 312032894, CU 338/1-2.)

A Spacetime Model

$$\mathfrak{M} = (\mathcal{M},\, g_{ab},\, T_{ab})$$

1. $\mathcal{M}$ is *any* 4-dimensional, Hausdorff, paracompact, connected manifold
2. g_{ab} is any Lorentz metric on $\mathcal{M}$
3. T_{ab} is such that

$$G_{ab} = 8\pi T_{ab}$$

where G_{ab} is the Einstein tensor of g_{ab}

What do people mean when they say such-and-such is a "solution" to the Einstein field equation?

(for the claim not to be trivial, there should be some $\mathfrak{M}$ that is not one)

generally, one of two things:

1. particular T_{ab} derived from Lagrangian based on Einstein-Hilbert action that satisfies other conditions (*e.g.*, minimal coupling)

2. T_{ab}, no matter how derived, satisfies further conditions (*e.g.*, the DEC)

NEITHER HAS ANYTHING TO DO WITH GENERAL RELATIVITY *PER SE*

you give me any Lorentz metric on any manifold,
I'll tell you what T_{ab} is

Psychology ("Physical Intuition")

you say:

- no "nice" field equations
- physically "unreasonable" in some sense
- ...

but that tells me only your favorite way of calling things "physical" (a fact about your psychology, not about the physical world)

(how can "physical intuition" be honed for regimes
we've never had experimental access to?)

all spacetime models

NO RESTRICTIONS!!!

What “Depends” on the Einstein Field Equation?

- geodesic theorems
- causality hierarchy
- singularity theorems
- black hole theorems
- Generalized Second Law
- positive energy theorems (ADM and Bondi)
- “conservation of matter” (T_{ab} vanishes on closed, achronal set $\Rightarrow$ vanishes in domain of dependence)
- entropy bounds
- Topological Censorship
- Cosmic Censorship Hypothesis
- Lorentzian splitting theorems
- a given globally hyperbolic extension of a spacetime is maximal
- CMC foliations for spatially compact spacetimes
- ...

none of them do!

in very strong sense. . .

assume logical negation of the Einstein field equation and theorems still hold

Actual Role of Einstein Field Equation:

provides physical interpretation

gives physical interpretation to conditions on "purely geometrical" object

logical form of arguments:

1. assume condition on some geometrical object (Riemann, Weyl, Ricci, . . .)
2. derive theorem in purely mathematical way
3. invoke EFE to give physical interpretation of condition assumed in step 1 and result derived in step 2

One Way to Make These Claims Precise and Vivid

- consider 4-d Einstein field equation as point-by-point relation
- fix manifold, impose Lorentz metric on part of it (say, open set): how to extend it?
- *only* imposition of conditions on stress-energy tensor or imposition of field equations on matter allows one to say what is "right" extension
- Einstein field equation alone constrains *nothing*

concrete example

1. fix favorite globally hyperbolic spacetime, and a Cauchy surface
2. slap conformal factor smoothly going from 1 at Cauchy surface to anything one wants to the future
3. still a globally hyperbolic spacetime
4. nothing in Einstein field equation as point-by-point relation among 4-dimensional tensors tells you which is "right" future

T_{ab} almost arbitrary, picking up terms like

- $\nabla_a \nabla_b \ln \Omega$
- $(\nabla_a \ln \Omega)(\nabla_b \ln \Omega)$
- $g_{ab}(\nabla^n \ln \Omega)(\nabla_n \ln \Omega)$
- $g_{ab} \nabla^n \nabla_n \ln \Omega$

globally hyperbolic is not sufficient
for "physically reasonable"

(3+1 canonical is not equivalent to general relativity even when restricted to globally hyperbolic spacetimes!)

General Relativity Is Not a Theory of a Particular Kind of Physical System

T_{ab} does *not* represent a matter field, rather a generic property all possible matter fields have (possible = "representable in general relativity") $\Rightarrow$ like a Hamiltonian or Lagrangian, not like a Maxwell field

Thus:

only each class of stress-energy tensors for a given type of matter field, or each class of matter field equations, forms its own physical theory

Thus:

general relativity is a framework within which one formulates physical theories, not a physical theory itself

(compare Newtonian, Lagrangian and Hamiltonian mechanics, Dirac formulation of quantum mechanics, algebraic quantum field theory, *etc*.)

If I am right. . .

"quantizing general relativity" is a non-starter: as in the move from classical to quantum field theory, different types of matter fields with different internal structure require or strongly suggest different quantization techniques