A Strengthened Zeroth Law for Black-Hole Thermodynamics

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Outline

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2. The Zeroth Law: A Serious Problem
3. Two Possible Characterizations of Mutual Equilibrium
4. Thermodynamics Regained, Maybe

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Fundamental Principles and Features of Standard Thermodynamics

not just four laws, but also:

- state space of equilibrium states
- temperature as mediator/measure of physical couplings
- classification of processes as adiabatic, reversible, and quasi-static
- important conjugacy relations between intensive and extensive variables
- additivity of entropy
- equilibrium states maximize entropy and minimize free energy
- the Clausius and Kelvin Postulates
- ...

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Zeroth Law

Thermodynamics The temperature T is constant throughout a body in thermal equilibrium.

Black Holes The surface gravity κ is constant over the event horizon of a stationary black hole.

REAL Zeroth Law

Two bodies in equilibrium with a third are in equilibrium with each other.

Fundamental Role of Transitivity

- construction of state space of equilibrium states
- state-function of intensive quantity physically identified as temperature
- equilibrium implies constancy of temperature
- temperature as mediator and measure of thermal coupling
- definition of entropy, its additivity
- characterization of processes as adiabatic, reversible, and quasi-static
- conjugacy of intensive and extensive quantities
- entropy maximization, free-energy minimization
- ...

constancy of temperature alone does not suffice

can we recover same for black holes?

insight either way. . .

differences point to need to modify statistical grounding, and place constraints on how to do it phenomenological thermodynamics is strongest evidence we have for character of underlying micro-dynamics whose statistical treatment gives rise to that thermodynamical behavior

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Black-Hole Equilibrium

stationary with respect to asymptotically timelike Killing field (null on event horizon): "stays the same with respect to fixed standard of time flow"

natural generalization

Mutual Equilibrium for Two Black Holes

both stationary with respect to same asymptotically timelike Killing field: “both stay the same with respect to the same fixed standard of time flow”

Problem

theorem: there are no physically reasonable solutions representing two stationary black holes

Solution

Approximate! black holes are both "approximately stationary" with respect to the same "approximate asymptotically timelike Killing field", null on their horizons

(thermodynamics is all approximation anyway: equilibrium is defined only for characteristic time scales, up to fixed levels of accuracy)

New Problem

What is an "approximate Killing field"?

"obvious" answers don't work:

- $\pounds_\xi \, g_{ab} \approx 0$
- $\sup_O \left\{ \left(g^{mn} \pounds_\xi g_{mn} \right)^2 \right\} < B$, for O compact closure and $B > 0$
- $\delta_\xi \int_O \left(g^{mn} \pounds_\xi g_{mn} \right)^2 = 0$
- ...

Solution: use *ad hoc* criterion that fits the physical structure of the problem. Two possibilities:

1. approximate constancy of temperature
2. approximate constancy of quasi-local mass

conjectures:

1. approximate constancy of quasi-local mass, with respect to surfaces tangent to almost-Killing field, implies almost-Killing field tangent to both horizons
2. approximate constancy of temperature and approximate constancy of quasi-local mass imply each other

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A Strengthened Zeroth Law for Black Holes?

Yes!

Claim: Constancy + Kelvin Postulate $\Rightarrow$ Transitivity

Recall:

The Kelvin Postulate

A transformation whose only final result is to transform into work heat extracted from a source that is at the same temperature throughout is impossible.

What Is "Mutual Equilibrium"?

claim: all we need is a necessary condition

Condition

when two bodies in mutual equilibrium are brought into contact, there will be no spontaneous transfer of heat from one to the other, and neither will spontaneously perform work on the other

Deriving the Zeroth Law (Crude Sketch)

three ordinary thermodynamical systems A, B and C

1. assume A and C in mutual equilibrium, and B and C, but not A and B
2. bring A and C into contact: no heat spontaneously transferred, so they are same temperature (similarly for B and C) $\Rightarrow$ A and B same temperature
3. bring A and B in contact
4. *ex hypothesi*, one must spontaneously perform work on the other
5. $\Rightarrow$ violation of the Kelvin Postulate
6. $\Rightarrow$ they are in mutual equilibrium

“Mutual Equilibrium” for Black Holes?

so: what are work and heat for black holes?

Free Energy, "Heat", and Work for Black Holes

- since M cannot be reduced below M_{irr} (irreducible mass):

$$\text{“free energy”} := M - M_{\text{irr}}$$

- so "quantity of heat" transferred in any process: change in total energy minus change in free energy:

$$\Delta Q_{\text{BH}} := \Delta M_{\text{irr}} \, (> 0)$$

- so "work": change in total energy not due to change in free energy (rotational and radiative processes)

The Kelvin Postulate for Black Holes

A transformation whose only final result is that a quantity of heat is extracted from a stationary black hole and transformed entirely into work is impossible.

Proof (Crude Sketch):

1. assume such a transformation possible
2. if heat extracted is purely gravitational, then change in irreducible mass must be strictly greater than change in total mass (*i.e.*, irreducible mass must increase)
3. so area (entropy) must also increase, violating the Postulate

(straightforward emendation to accommodate heat transfer by Hawking radiation: the irreducible mass of the black hole will still necessarily change, implying change in entropy)

“Mutual Equilibrium” for Black Holes

Two black holes are in mutual equilibrium when they are each stationary with respect to the same timelike Killing field.

Promissory Note

construction of equilibrium state-space, *etc.*

Problem

no known exact solutions (without naked singularities) representing "multiple Kerr black holes", so:

- need to work with approximation solutions
- need to characterize "approximate Killing field"

Perhaps not so bad...

ordinary thermodynamics is all approximative anyway; "equilibrium" always has to be characterized with respect to a relevant timescale; but still a non-trivial problem for black-hole thermodynamics to be resolved