Singularities in Classical and Semi-Classical Gravity—Theory Failure or Groovy New Physics?

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Erik Curiel

Munich Center For Mathematical Philosophy
Ludwig-Maximilians-Universität

Black Hole Initiative
Harvard University

erik@strangebeautiful.com

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Outline

Propædeutic to Singularities in General Relativity

Cosmic Censorship and Determinism

Singularities and Cosmic Censorship in Semi-Classical Gravity

Breakdown of Theory or Groovy New Physics?

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Breakdown of Theory or Groovy New Physics?

Incomplete Worldlines

- sometimes worldlines "just end" after a finite period of time
- cannot extend them in the spacetime, cannot extend spacetime itself
- no event "at which they terminate": breakdown of spacetime geometry?
- observer would experience only finite length of time of existence without ever dying or ceasing to exist in any way

Singular Structure: Canonical Definition

Definition

in a spacetime satisfying a maximality condition: an inextendible causal curve incomplete with respect to some measure of "length"

places almost no substantive constraint on nature or physical properties of spacetime

Examples

- delete from spacetime set in shape of Ingrid Bergman—but seems like cheating
- blow-up of spacetime curvature in gravitational collapse: metric "loses definition at $r = 0$"
- some incomplete, inextendible curves in spacetimes with no surgery and no pathology in curvature (Taub-NUT, colliding plane gravitational waves, ...)
- generic features of otherwise "well behaved" spacetimes (Penrose, Geroch, and Hawking singularity theorems)

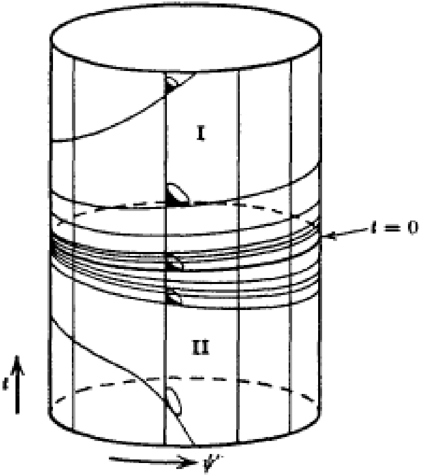

Figure: Misner's 2-d Taub-NUT (taken from Hawking and Ellis 1973)

incomplete, inextendible null geodesics "bunch up" at $t = 0$, but timelike curves pass through them without problem

Physical Problems

- general relativity is time-reversible: worldline ("observer") could just "pop into existence" right in middle of spacetime ("Principle of Sufficient Reason"!)
- any physical mechanism that could cause such a thing, going forward or backward in time?
- how to formulate laws when spacetime geometry loses definition (curvature pathology)?
- physical significance of curvature pathology? (often ambiguous, depends on motion of observer probing spacetime)
- spacetimes can be timelike-complete but null-incomplete, and vice-versa: how to understand this physically?
- spacetimes can be geodesically complete, but have incomplete timelike curves of bounded total acceleration

(Earman 1995; Curiel 1999)

Philosophical Problems

- plethora of definitions (canonical not only one): such a thing as "the single, correct definition"?
- if not, what is relation among all the viable definitions? are they different explications of the same concept?
- ambiguity, meaning of different notions of "length" used to determine incompleteness
- choice of reference class of spacetimes to define "maximality" (Manchak 2016, 2017; Doboszewski 2017, 2019)
- "to exist is to exist in space and time": incompleteness of curves don't—what sort of existence to attribute to them?
- breakdown of determinism?
- breakdown of general relativity as physical theory?
- what to go on besides intuitions of eminent physicists?

(Earman 1995; Curiel 1999)

Psychological Problems: The Finitude of Existence

The mind of man, by nature a monist, cannot accept two nothings; he knows there has been one nothing, his biological inexistence in the infinite past, for his memory is utterly blank, and that nothingness, being, as it were, past, is not too hard to endure. But a second nothingness—which perhaps might not be so hard to bear either—is logically unacceptable.

<u>Ada</u>
V. Nabokov

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the immodesty of naked singularities

Penrose (1969):

> *Even if [the No-Hair Theorem], or something like it, is true, have we any right to suggest that the only type of collapse which can occur is one in which the space-time singularities lie hidden, deep inside the protective shielding of an absolute event horizon? . . . . [If not, it would be] possible for information to escape from the singularity to the outside world. . . . In short, the singularity is visible, in all its nakedness, to the outside world! . . . .*
>
> *We are thus presented with what is perhaps the most fundamental unanswered question of general-relativistic collapse theory, namely: does there exist a "cosmic censor" who forbids the appearance of naked singularities, clothing each one in an absolute event horizon?*

cosmic censorship can’t just be “no naked singularities”

there’s one in our universe!

the big bang

two responses

1. refine what one means by "naked singularity"
2. provide necessary and sufficient conditions for "cosmic censorship" in general, independent of the idea of a naked singularity

modern impetus

ensure determinism/predictability, “physics as we know it”

a breakdown in the fundamental structure of spacetime so severe that it could wreak havoc anywhere it were visible

- the structures that break down are required for the formulation of known physical laws
- and in particular for initial-value problems

$\Rightarrow$ determinism and predictability, perhaps the possibility of physics as we know it, would collapse wherever the pathology were causally visible

but what determinism?

there are many forms of "determinism", and they can all fail in different ways

(Earman 1995; Doboszewski 2017, 2019)

well-posed Cauchy problem

example: one standard "strong" formulation

The maximal globally hyperbolic future developments of generic asymptotically flat initial data are future inextendible to regular Lorentzian manifolds.

very popular!

BUT:

- anything remotely resembling a proof depends on assuming auxiliary conditions (*e.g.*, energy or causality conditions) that themselves are as weakly supported or otherwise problematic as cosmic censorship itself
- problems with ambiguity of "inextendibility of spacetime" (Manchak 2016, 2017; Doboszewski 2017, 2019)
- problems with making "generic" well defined and physically significant (Curiel 2015)
- counter-examples have been found to essentially every precisely formulated proposal along those lines (and all others), leading to *weak* current formulations (*e.g.*, ruling out only *discontinuous* extensions!)

several open and serious conceptual problems

1. why focus on indeterminism rather than other forms of "pathology"?
2. 3+1 $\neq$ 4 (canonical formulation – evolved globally hyperbolic – is not general relativity)
3. there are *many* different formulations like this, none equivalent, all capturing subtley different conceptions, all with their own conceptual virtues and demerits—which to choose? (Earman 2020, personal communication)
4. why expect that one principle will cover all important cases?
5. what justifies the urge to find such a thing in the first place?

why one principle?

Why try to find *The* Principle of Cosmic Censorship, rather than several different propositions that make precise and capture different aspects of what we roughly and crudely have an idea we want, each relevant in its own context but none applicable universally?

An unrepentant and degenerate old Aristotelian essentialism, I think, that *demands* a single definition for what we use a single name for.

why look for it in the first place?

What justifies the urge to find such a thing in the first place? What reasons may we have beyond the brute deliverance of entrenched physical theory—which we don't have—to expect such a proposition may hold?

three kinds of physical principles

1. those required by the physics (for any of several reasons)
2. those required on pragmatic grounds (any of several)
3. those desired for psychological comfort

required by the physics

1. conflicts with another *prima facie* fundamental, entrenched principle (conservation of energy, *e.g.*)
2. blocks theoretical development (quantum gravity, *e.g.*)
3. ...

required on pragmatic grounds

1. conflicts with how we like to "do" physics (predictability based on IVPs, *e.g.*)
2. makes modeling systems difficult
3. ...

desired for psychological comfort

1. conflicts with how we like to "do" physics (predictability based on IVPs, *e.g.*)
2. aesthetic grounds
3. fear of unpredictability
4. neurotic Cartesian phobia of uncertainty
5. ...

the demand for cosmic censorship, as the ethically evaluative flavor of its very name suggests, and as the connection with determinism shows, arises from both pragmatic and psychological grounds

not physical grounds

in sum, we have. . .

an example of a sizable subculture in physics working on matters that:

- have no clearly or even unambiguously defined physical parameters to inform the investigations
- have no empirical evidence to guide or even just constrain them
- have the parameters of the debate imposed by and large by the intuitions of a handful of leading researchers

From sociological, physical, and philosophical vantage points, one may well wonder why so many physicists continue to work on it, and what sort of investigation they are engaged in.

52 years of Cosmic Censorship…

and still we do not know what it is or why we should care this is not to say that mathematical work on the initial-value formulation is not interesting and important in and of itself—only that it is not clear to me that the attempt to "resolve the problem of cosmic censorship" ought to frame and motivate that work

cf. the, to my mind, unhealthy obsession with differentiability class of extensions—can it possibly make a *physical* difference whether a metric is $C^{1,1}$ but not C^2 extendible, to take what is these days a tame example, especially given that no one expects general relativity to be a "final" theory anyway?

semi-classical Einstein field equation (SCEFE)

$$G_{ab} = 8\pi \langle \hat{T}_{ab} \rangle$$

(classical Einstein tensor =
expectation value of stress-energy tensor as quantum operator)

many fascinating mathematical, physical, conceptual, philosophical problems…

which we, sadly but expeditiously, here ignore semi-classical gravity promiscuously, with joyous abandon, violates every energy condition under the sun—whither the singularity theorems?

Theorem (Wall 2013)

Suppose there exists a globally hyperbolic spacetime with a quantum trapped surface T *[i.e., a certain kind of null surface with semi-classical entropy decreasing to the future]. Let the semiclassical approximation be valid near* T *(but not necessarily elsewhere). Then the fine-grained Generalized Second Law requires that spacetime is not null-geodesically complete.*

1. Bousso and Engelhardt (2016): evaporating black holes have trapped surfaces outside of horizons
2. Engelhardt and Folkestad (2020): in holographic spacetimes, trapped surfaces lie inside horizons (but their assumptions are suspect)
3. Fewster and Kontou (2020): like classic singularity theorems, but with weakened energy conditions that we expect to hold for evaporating black holes

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Role of Einstein Field Equation?

for all classical singularity theorems, I claim:
none depends on it!

in very strong sense...

assume logical negation of the Einstein field equation and theorems still hold (when proven)

Wall's theorem may depend on the SCEFE—it's a subtle issue, depending on how one interprets the GSL

Actual Role of Einstein Field Equation:

provides physical interpretation

gives physical interpretation to conditions on "purely geometrical" object

logical form of arguments:

1. assume condition on some geometrical object (Einstein, Ricci, . . .)
2. derive mathematical theorem
3. invoke EFE to give physical interpretation of condition assumed in step 1 and result derived in step 2

if singularity theorems do not depend on Einstein field equation, in what sense can they mark a breakdown of *general relativity* itself?

- what actually "breaks down", if anything, is the "idea" of Lorentzian geometry, not general relativity itself
- but it is difficult to make this precise: a singular space-time, after all, is *everywhere* a well defined Lorentzian manifold
- illuminating comparison: divergence of kinetic energy times in finite time in solutions of Navier-Stokes equations—that *really is* a breakdown of the terms of the theory, for the KE terms become ill-defined at a determinate point of space at a determinate instant of time

Of course, general relativity *depends on* Lorentzian geometry, but the "breakdown" here, if any, does not come from the Einstein field equation itself, but rather generic features of Lorentzian geometry, such as the Raychaudhuri equation.

So the breakdown, if there is one, seems to be:

- conditions for the definition of structures required for the formulation of the theory no longer apply
- or, perhaps: since the Einstein field equation only gives physical interpretation to conditions on Ricci tensor, breakdown is in possibility of giving physical interpretation to geometrical structures in the conceptual terms of the theory
- when *Ricci* curvature pathology is involved, that may mark a breakdown of the theory (or theories) of *matter* at issue, but, again, seemingly not general relativity

But keep in mind: singularities do not even herald the breakdown of the theory, in any straightforward sense, because the Lorentzian geometry is always, almost by definition, *well defined at every point of the spacetime manifold*.

That various measures of curvature (or derivatives of curvature) grow without bound along certain “short” curves may bother us psychologically, and may even be wrong as a prediction, but that does not necessarily mean the theory has “broken down” in any interesting sense. It just made a bad prediction.

Now, it may be that this particular bad prediction (if it be one) is fruitfully understood as indicating a regime where the effects of phenomena not treated by general relativity (*e.g.*, "quantum" or "thermodynamically non-equilibrium" behavior of some geometrical structure) become non-negligible compared to the scales at which general relativity is appropriately applied and predictively adequate. But it may not.

Compare Newtonian mechanics applied to bodies with arbitrarily high velocities. I think this is not fruitfully understood as a case in which the theory "breaks down", even though some quantities, *e.g.*, linear momentum and kinetic energy, grow without bound. I think it is rather that the theory makes bad predictions, and we now have one that does a better job.

Briefly, then, what I mean by "breakdown" here is: the theory is no longer appropriately applied, *i.e.*, it fails to have the conceptual or mathematical resources needed to construct a model of the system at issue that can be checked for predictive accuracy.

If singularities herald a breakdown of general relativity, then it means that, as a theory, it does not have the conceptual or mathematical resources needed to model the relevantly "quantum" or "thermodynamically non-equilibrium" behavior that, *ex hypothesi*, manifests in regions where general relativity predicts strong curvature. But that has nothing to do with singularities.

N.b.: if classical "curvature blowup" heralds the need to use funky quantum structures to model a "region of spacetime", then *ipso facto* any classical curve that enters that region will be "incomplete and inextendible" according to general relativity...

so what exactly does is this promise of "singularity resolution" by quantum gravity anyway?

so maybe there really are “naked singularities”, and they spew out old socks and TVs playing Nixon’s “Checkers Speech”...

and if so, then we know that their behavior will not be governed by general relativity—but we could then begin to investigate their behavior as we would any other initially strange physical system, to see whether we can find laws that describe them

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