The Idiosyncracy of the Generalized Second Law

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Outline

Now the rainman gave me two cures
Then he said, "Jump right in"
The one was Texas medicine
The other was just railroad gin
An' like a fool I mixed them
An' it strangled up my mind
An' now people just get uglier
An' I have no sense of time
Oh, Mama, can this really be the end
To be stuck inside of [a black hole]
With the [entropy and GSL] blues again

– Bob Dylan
"Stuck inside of Mobile with the Memphis Blues Again"

The law that entropy always increases,—the second law of thermodynamics—holds, I think, the supreme position among the laws of Nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations—then so much the worse for Maxwell's equations. If it is found to be contradicted by observation—well, these experimentalists bungle things some times. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

– Arthur Eddington (1935)
The Nature of the Physical World

Bekenstein's original motivation (pre-Hawking):

TO SAVE THE SECOND LAW!

seems easy to violate standard Second Law when black holes are around:

1. throw favorite highly entropic system (Wheeler's tea cup) into black hole
2. entropy of world outside event horizon—a causally isolated system—spontaneously decreases
3. event horizon does not make the cup's entropy practically inaccessible, as would happen if we launched it far away with a rocket
4. but *in principle* inaccessible to the outside
5. and then it would vanish entirely in the interior singularity
6. BHs, it would seem, are entropy decreasers *nonpareil*
7. and so the Second Law, that most cherished of all physical principles, would follow the cup into non-being, and so (per Eddington) BHT itself

⇒ Bekenstein (1972, 1973, 1974) proposed Generalized Second Law:

total entropy, black hole (area) + ordinary matter outside, never decreases:

$$\boxed{\frac{\delta A}{4} + \delta S_{out} \geq 0}$$

- for Bekenstein, BH entropy was measure of information about BH interior inaccessible to an exterior observer
- he argued we need to attribute entropy to event horizon itself, to save Second Law outside, because "we can't know that the entropy inside isn't really decreasing"
- $\Rightarrow$ extraordinary physical insight and understanding, great theoretical advance
- BUT: based on bad arguments (even putting the information-theoretic mess aside)! (Curiel 2019; Wüthrich 2019)
- marvelous example of how bad arguments can still lead to profound illumination; happens all the time in physics (and mathematics and philosophy!) in pioneer work

same claim, better arguments

Unruh and Wald (1982):

1. can seemingly violate the Kelvin Postulate using Geroch's infamous example of lowering a box of radiation to the event horizon
2. taking account of buoyancy of box "floating" in acceleration radiation near event horizon (Unruh effect) preserves Kelvin Postulate
3. $\Rightarrow$ QFT-CST saves the day!
4. $\Rightarrow$ seemingly need *all* of BHT (Zeroth Law, First Law, Hawking effect, GSL, ...) to ensure its internal consistency

Wall (2009, p. 2):

> *Here a 'proof' means a detailed argument trying to establish the GSL for a broad range of states in some particular regime.*

issues to settle:

1. what BH entropy to choose?
   1.1 Gibbs: Mukohyama (1997) and Sorkin (1998)
   1.2 Boltzmann: Wald (1976)
   1.3 information-thoretic: Bekenstein (1974, 1975)
   1.4 entanglement: Zurek and Thorne (1985) and Thorne et al. (1986)
   1.5 holographic: Wall (2015) and Engelhardt and Fischetti (2019)
   1.6 Noether: Wall (2015)
2. what matter entropy to choose? (only first four of previous list are viable)
3. what horizon to choose? not every null surface is appropriate[1]
   3.1 standard event horizons: most
   3.2 apparent horizons: Thorne et al. (1986) and Engelhardt and Wall (2018a)
   3.3 dynamical trapping horizons: Hayward (1994) and Kurpicz et al. (2021)
   3.4 cosmological BH horizons (dS/AdS): He and Zhang (2007), Zhou et al. (2007), and Hu et al. (2019)
   3.5 quantum extremal surfaces (QES): Engelhardt and Wall (2015) and Engelhardt and Wall (2019)

1. See Wall (2009, p. 4) for an explanation of why, *e.g.*, the GSL cannot apply to a spherically symmetric trapped surface inside a Schwarzschild black hole.

issues to settle (cont.):

4. what regime to choose?
   - 4.1 for Wall (2009, §1.2), choice depends on "what restrictions the proof needs to impose on the perturbations of the black hole" ⇒ two questions:
     - 4.1.1 "how large and how rapidly changing these perturbations are allowed to be"
     - 4.1.2 "how many features of quantum mechanics are taken into account"
     - 4.1.3 more needs to be fixed, including how the BH is defined in first place (intimately related to what sorts of perturbation to consider)
   - 4.2 Wall (2009, §1.2) then uses first question to distinguish what he calls:
     - 4.2.1 *quasi-stationary*: allow small but otherwise arbitrary perturbations to stationary background metric
     - 4.2.2 and *quasi-steady*: matter fields in approximately steady state with respect to Killing field generating the horizon, over periods of time on order of BH radius
     - 4.2.3 *adiabatic*: what one means by "adiabatic" in this context is delicate and subtle issue; would take us too far afield; see Wall (2009, p. 7*ff*.)

     quasi-steady implies, but is not implied by, quasi-stationary; quasi-steady also implies the First Law of BHM
   - 4.3 I would add: regime in which *near-horizon* effects can be ignored (necessary for some S-matrix proofs, *e.g.*, Frolov and Page 1993)
   - 4.4 then he uses the second question to define 4 further regimes:
     - 4.4.1 classical BHT
     - 4.4.2 hydrodynamic approximation
     - 4.4.3 semi-classical regime
     - 4.4.4 full QG

issues to settle (cont.):

5. what spacetime structure to choose (shape and character of spacetime)?
   5.1 exact solution: Schwarzschild, Kerr, Reissner-Nordström, Kerr-Newman, dS, AdS, dS-Schwarzschild, AdS-Schwarzschild, ...
   5.2 abstract characterization:
      5.2.1 type of horizon: event, isolated, trapping, cosmological, general causal, QES, ...
      5.2.2 eternal, past horizon, stationary, quasi-static, dynamic
      5.2.3 topology: form of domain of outer communication (if a black hole); ...
      5.2.4 symmetries
      5.2.5 other asymptotic structure, *e.g.*, some form of flatness or predictability
6. character of quantum fields?
   6.1 QFT formulation (S-matrix, algebraic, canonical based on a Lagrangian, holographic, low-energy quantum gravity, ...)
   6.2 flavor of QFT (scalar, vector, bosonic, fermionic, ...)
   6.3 choice of eigenbasis needed? if so, which? or generic conditions imposed?
   6.4 choice of state needed? if so, which? or generic conditions imposed?
   6.5 boundary conditions
7. auxiliary conditions:
   7.1 energy conditions
   7.2 entropy conditions
   7.3 cosmic censorship (but see Hod 2020)
   7.4 causal assumptions (*e.g.*, chronology)
   7.5 topological assumptions (*e.g.*, topological censorship)
   7.6 stability assumptions ("small perturbations do not destroy the event horizon")

generic structure of proofs

none!

they're all over the place

indeed, there are now, at a conservative estimate,

$$\boxed{234,125,625,984}$$

possible derivations[2]

2. with a tip of the hat to I. J. Good (1971)

Wall (2009, p. 2):

> *Most of the proofs are unsound. Some have inconsistent or erroneous assumptions, and others have hidden gaps in the reasoning. Nevertheless each of these proofs is valuable. Even an invalid proof can clarify the issues and choices that must be resolved in order to fully understand the GSL. Faulty proofs might also be correctable through small adjustments. It is better to view them as research programs than as mere fallacies.*

Emden (1938, p. 909):

As a student, I read with advantage a small book by F. Wald entitled 'The Mistress of the World and her Shadow'. These meant energy and entropy. In the course of advancing knowledge the two seem to me to have exchanged places. In the huge manufactory of natural processes, the principle of entropy occupies the position of manager, for it dictates the manner and method of the whole business, whilst the principle of energy merely does the bookkeeping, balancing credits and debits.

1. What is the relationship between the seemingly fundamental physical principles known as "energy conditions", both in classical GR and in SCG, and the GSL?
2. In what sense can one derive energy conditions from the GSL and vice-versa, and what does that tell us about the relationship between energy and entropy in SCG?

energy conditions in classical GR:

- constraints on measure of curvature (usually Ricci or Einstein), given physical interpretation by way of stress-energy tensor in EFE
- EFE on its own has almost no physical content without them
- almost every general, deep result in GR assumes an energy condition for its proof. . .
- by way of guaranteeing focusing of geodesics

most characteristic role of energy in GR also most characteristic of energy conditions:

- in classical GR, as in most theories, one has a great deal of freedom in what one takes as primitive and what as derived
- think of the geodesic principle and covariant conservation of stress-energy, inter-derivable
- this is not true of the classical energy conditions, neither pointilliste nor impressionist
- one *can't derive energy conditions* in classical GR
- they are always taken as primitive (Curiel 2017)
- perhaps they reach down to and get ahold of spacetime structure at a very deep level?

- this is not so of *entropy* conditions in classical GR
- Bousso (1999a, 1999b), *e.g.*, used the DEC in his original work to motivate his covariant entropy bound ("the total entropy flux S_L through any null hypersurface L satisfying some natural geometrical conditions must be such that $S_L \leq A/4$, where A is a spatial area canonically associated with L")
- Flanagan et al. (2000) then proved it using the NEC

the relations between energy and entropy are neighborly, but not intimate:

1. relation between energy conditions and entropy conditions is "one way" only
2. there is no explicit unification of different types of entropy (in something like a GSL, *e.g.*), as there is for energy in $\nabla_n T^{an} = 0$
3. as in classical thermodynamics—if one accepts the orthodox dogma (Wald's Way), that there is no consistent thermodynamical theory of purely classical black holes—energy and entropy are not jointly fungible (throwing mass into a classical black hole doesn't increase its entropy)
4. there is still no entropometer
5. and relation of *both* stress-energy and entropy to equilibrium (existence of timelike Killing field) is obscure at best

in any event, already energy here goes beyond the role it plays in non-relativistic physics...

to paraphrase Emden's marvelous remark, in the huge manufactory of natural processes, energy begins to occupy the position of, if not manager, at least assistant manager, for it constrains the manner and method of the whole business, in conformance with the constraints already imposed by entropy

How did classical general relativity know that the horizon area would turn out to be a form of entropy, and that surface gravity is a temperature?

Ted Jacobson
"Thermodynamics of Spacetime:
The Einstein Equation of State"

first hints of more intimate relations between energy and entropy in SCG:

fungibility energy can now be directly transformed into entropy ("throw stuff into black hole"), and vice-versa (Hawking radiation); each is a direct measure of the other ("area and mass tell you each other")

zero point they both have natural zero points, which is the same state (Schwarzschild $M = 0$)

equilibrium heuristic but compelling arguments that stationary black holes minimize one form of free energy ("$M - M_{\text{irr}}$") and maximize entropy

free energy unlike in ordinary thermodynamics, the "Helmholtz free energy" of BHs ($M - S_{\text{B}}T_{\text{H}}$) always decreases as entropy ($A/4$) increases[3] (Sorkin 1998)

entropometer! we can measure area of event horizon directly (not a modal quantity)—which is also, in this case an ergometer, as area gives you mass

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3. Since the First Law for BHs has no pressure-volume term, the Helmholtz and Gibbs free energies coincide.

but the relations—including equalities—between entropy and energy even beyond those just stated become now a promiscuous, libertine, orgiastic debauch. . .

- one of the most natural ways to think of gravitational energy in GR (yet largely unexplored) is by way of geodesic deviation (focusing) (Penrose 1966; Curiel 1997)
- one can extract energy from a gravitational field when and only when there is geodesic deviation (focusing) (Bondi and McCrea 1960; Curiel 1997)
- as we'll see, entropy now becomes associated with geodesic deviation (focusing) as well

entropy conditions[4] take on the classical role of energy conditions, by guaranteeing geodesic focusing:

1. GSL proves a singularity theorem, and rules out traversable wormholes, negative masses, other forms of faster-than-light travel between asymptotic regions, restarting inflation and CTCs: Wall (2013)
2. quantum Penrose inequality using generalized entropy of bulk light sheets to constrain lower bound of ADM mass: Bousso et al. (2019)
3. Bousso bound proves a singularity theorem: Bousso and Shahbazi-Moghaddam (2022)
4. quantum focusing implies singularity theorems, the GSL and boundary causality: Shahbazi-Moghaddam (2022)

(*N.b.*: the last only a conjecture, with supporting plausibility arguments and evidence from test cases)

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4. It is almost wholly unclear what is meant by entropy in any given application, and whether, in any event, they are jointly consistent.

the principle of entropy increase (GSL) becomes fecund!

one now has, for the first time, not only derivations of energy conditions, but ones *based on entropy conditions*, and vice-versa:

1. proving the (A)ANEC from the GSL: Wall (2010)
2. proving the ANEC from the QNEC: Bousso et al. (2016)
3. proving the NEC from the GSL: Parikh and Svesko (2017)

(we'll come to the QNEC—Quantum Null Energy Condition—next)

QNEC, Bousso et al. (2016):

1. any point p and null vector k^a define (at least locally) a null plane N
2. given any codimension-2 surface Σ that contains p and lies on N, consider the von Neumann entropy S_{out} of the quantum state of the ambient quantum fields restricted to one side of Σ
3. a second variation S''_{out} can be defined by deforming Σ along N, in a small neighborhood of p, by an area $\mathcal{A}$
4. QNEC:

$$\boxed{\langle T_{kk}(p)\rangle \geq \frac{\hbar}{2\pi} \lim_{\mathcal{A}\to 0} S''_{\text{out}}/\mathcal{A}}$$

Figure

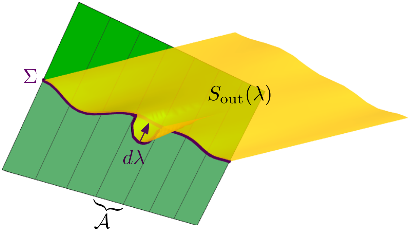

FIG. 1. The spatial surface Σ splits a Cauchy surface, one side of which is shown in yellow. The generalized entropy S_{gen} is the area of Σ plus the von Neumann entropy S_{out} of the yellow region. The quantum expansion Θ at one point of Σ is the rate at which S_{gen} changes under a small variation $d\lambda$ of Σ, per cross-sectional area $\mathcal{A}$ of the variation. The quantum focusing conjecture states that the quantum expansion cannot increase under a second variation in the same direction. If the classical expansion and shear vanish (as they do for the green null surface in the figure), the quantum null energy condition is implied as a limiting case.

(shamelessly cribbed from Bousso et al. (2016))

one moreover gets *equivalences* of entropic and energetic quantities:

1. Leichenauer et al. (2018): for null shape deformations as they appear in the QNEC, modulo a plausible, supported conjecture, second variations of the von Neumann entropy determine the full stress-energy tensor expectation value as an equality (and so, *à la* Jacobson 1995, 2016, one gets the EFE)
2. Wang (2020): (quasi-local) Bartnik-Bray inner mass exactly equals the (generally non-local) irreducible mass corresponding to the (generally non-local) outer entropy (Engelhardt and Wall 2018b)

perhaps most striking (argument due to Manus Visser, personal correspondence):

1. set
$$\langle K_\xi \rangle := \int_\Sigma \langle T_m{}^n \rangle \xi^m \mathsf{d}\Sigma_n$$
2. then SCG First Law:
$$\delta M = \frac{\kappa}{8\pi}\delta A + \delta\langle K_\xi \rangle$$
3. invoke First Law of quantum thermodynamics, *a.k.a.*, First Law of entanglement
$$\delta\langle K_\xi \rangle = T_{\mathsf{H}}\delta S_{\mathsf{ent}}$$
where $\delta S_{\mathsf{ent}} = -\operatorname{Tr}\rho\log\rho$ and $\rho = \frac{1}{Z}e^{-\beta_H K_\xi}$
4. $\Rightarrow$
$$\boxed{\delta M = T_{\mathsf{H}}\delta S_{\mathsf{gen}}}$$
where $S_{\mathsf{gen}} = A/4 + S_{\mathsf{ent}} = S_{\mathsf{B}} + S_{\mathsf{ent}}$

S_{gen} obeys *both* a First Law and a Second Law!!!

Now when the appearance of one thing is strictly connected with the appearance of another, so that the amount which exists of the one thing depends on and can be calculated from the amount of the other which has disappeared, we conclude that the one has been formed at the expense of the other, and that they are both forms of the same thing.

– James Clerk Maxwell
The Theory of Heat (ch. iv, p. 93)

Are energy and entropy different aspects, different forms, of the same underlying entity? Should this be one of the unifications we seek now in physics?

questions

1. the Clausius and Kelvin Postulates?
2. violation of standard energy conditions?
3. adding entropies of different grain?
4. relation of isentropic and reversible?
5. why a classical proof?
6. the fecundity of the GSL?
7. new arrow of time?

Clausius and Kelvin Postulates

Postulate (Lord Kelvin)

A process whose only final result is to transform into work heat extracted from a source that is at the same temperature throughout is impossible.

Postulate (Clausius)

A process whose only final result is to transfer heat from a body at a given temperature to a body of a higher temperature is impossible.

- Clausius and Kelvin Postulates are true formulation of classical Second Law (Fermi 1937; Curiel 2014)
- used, *inter alia*, to prove Carnot's Efficiency Theorem, ground of definition of absolute temperature
- *not* principle of entropy non-decrease
- can we formulate semi-classical versions? if so, prove them?
- if not, given central and fundamental role of the Postulates in ordinary thermodynamics, what can or ought this tell us about modifications required for thermodynamics and SM of BHs?
- Curiel (2014) proposes formulations for classical BHs, gives plausibility arguments, all easily extendible to SCG—need more precise formulations and rigorous proofs (or, at least, "proofs" in the sense of Wall 2009)

Hawking radiation necessarily violates null energy condition (NEC)

BUT—*essentially all* GR black hole theorems used in BHT (including those used in essentially all proofs of the GSL) depend on NEC

1. No Hair theorem
2. the event horizon of a stationary black hole is a Killing horizon
3. Zeroth and Third Law of black hole mechanics
4. positivity of ADM and Bondi masses
5. if T_{ab} vanishes on a closed, achronal set, it vanishes in the domain of dependence of that set ("conservation of vacuum")
6. formation of trapped surface after gravitational collapse
7. black holes are (topologically) spherical
8. black holes don't bifurcate
9. apparent horizons hidden behind event horizons
10. domain of outer communication is topologically simple
11. Bousso's covariant entropy bound
12. asymptotically flat spacetimes without naked singularities are asymptotically predictable
13. many standard general forms of cosmic censorship
14. . . .

what can we trust, and why?

differently grained entropies

$$\boxed{S_{\text{gen}} := \frac{A}{4} + S_{\text{m}}}$$

- in almost all proofs, S_{B} $(= A/4)$ is treated as a coarse-grained entropy (Engelhardt and Wall 2018a), S_{out} as a fine-grained one—does it make physical sense to add them?
- in any event, weird to add them in the first place—do we do this anywhere else in physics?
- and even weirder that it grounds the GSL

isentropic and reversible

- contrary to ordinary thermodynamics, in BHT the idea of a process's being isentropic and being reversible come apart (Christodoulou 1970; Curiel 2014)
- whence? why? how? whither?

classical proof?

- why does the GSL admit of classical proof *in vacuo* (Area Theorem), when, as it is argued, BHT in general and the GSL in particular require the Hawking effect—an indubitably quantum phenomenon—for its internal consistency and cogency (Unruh and Wald 1982, *e.g.*)?
- as Jacobson plaintively asks, how does classical general relativity know that the horizon area would turn out to be a form of entropy?

fecundity

- why does GSL imply non-trivial, indeed deep and general propositions when classical principle of entropy non-decrease does not?
- what can this tell us about the Second Law as extended into these new regimes with the attendant modifications?
- how does coupling *quantum* matter with gravity render the Second Law physically more substantive?
- is this epistemic sterility of ordinary Second Law tied up with its modal character?

gravitational arrow of time

idea of gravitational arrow of time highlights fact that time in GR, I think we often forget or neglect, is a *statistical* quantity for any extended body

1. each muscle fiber, cell, molecule, atom, . . . , in my body has its own world line (in relevant approximation where all treated as point particles)
2. each with its own proper time
3. which differs from the rest, even if only by relatively infinitesimal amount
4. and so the "proper time" of my body *en bloc* is itself only some kind of statistical averaging of those proper times

- utterly negligible in most cases of physical interest, even in astrophysics and cosmology
- BUT should become of possible relevance in *some* cosmological and astrophysical contexts, such as very early universe and near-horizon accretion disks around black holes, where constituents move at relativistic speeds
- what kind of statistical averaging is appropriate here? (would be excellent MA research project!)
- issue gets exponentiated when considering possible "gravitational micro-degrees of freedom", I should think
- does this make the connection between time and entropy, and so the Second Law, more intimate?
- in any event, this marks profound difference between time and space in GR: spatial position is not similarly statistical

- physical entropy has heretofore been attributed to *material* systems with non-trivial *dynamics*
- naively, spacetime geometry ("gravity") seems radically different from matter
- $\Rightarrow$ does black hole thermodynamics, and in particular gravitational entropy, militate in favor of effacing this difference?
- is there a principled difference between matter and geometry/gravity at small enough scales?

one fundamental and characteristic property of "matter":

it possesses stress-energy
as represented by a T_{ab}

the thermodynamical fungibility of stress-energy

Ground of First Law of Thermodynamics:

all forms of stress-energy are in principle ultimately fungible—any form of stress-energy can in principle be transformed into any other form

- the family of all T_{ab} has a natural linear structure
- moreover, all stress-energy tensors must have the "physical dimension of stress-energy"
- $\Rightarrow$ the physical meaning of being able to add them together

"gravity has no stress-energy tensor,
so it can't be matter"

but not so quick:

- sometimes possible to attribute *non-local* energy-like quantities, *i.e.*, not representable by a stress-energy tensor
- *e.g.*: gravitational radiation, ADM mass, various quasi-local masses, *etc.*
- so this criterion is not so clear
- anyway, other forms of energy in other theories are non-local (heat, work, Newtonian gravitational energy, ...)

another try: EFE contains only contributions from "matter" stress-energy, so in general relativity another "obvious" answer:

$\Rightarrow$ matter is Ricci tensor and gravity is Weyl tensor

BUT:

- in many spacetimes (*e.g.*, generalized FLRW: Carlo Alberto Mantica 2016) matter directly includes Weyl contributions
- and conditions on Weyl tensor determine form of the matter
- anyway, gradient of matter T_{ab} depends on divergence of Weyl ("Lanczos tensor")
- $\Rightarrow$ Weyl versus Ricci doesn't make the right distinction

AND now in SCG:

- in classical general relativity, "matter can transform into curvature" (gravitational collapse into a singularity)
- in black hole thermodynamics, "curvature can transform into matter" (Hawking radiation)

$\Longrightarrow$ breakdown of distinction between "matter" and "geometry"? requires radical changes to picture of ontology of spacetime and matter?

so, speculatively:

- matter and geometry not truly independent
- but different "manifestations of underlying unified entity"
- compare electric and magnetic fields in Maxwell theory, time and space in special relativity, ...

(seems to be suggested by some programs of quantum gravity)

Russell (1927, part i, ch. xiii, pp. 121–122):

> *To the philosopher, the difference between "matter" and "empty space" is, I believe, merely a difference as to the causal laws governing succession of events, not a difference expressible as that between the presence or absence of substance, or as that between one kind of substance and another.*

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