

What to Do When You Can't Solve Equations

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Patton, "Fishbones, Wheels, Eyes, and Butterflies:..." (p. 4):

The structuralist, structural realist, and methodological realist accounts all rely on one thing: a history of successful scientific theories in which not only posits about entities, but also formal elements like variables and relations, are indispensable instruments.

I want to begin by raising the question: what happens, then, if precisely the problem that faces scientists is the inability to solve the equations, the inadequacy of our current theory to provide a model of a given domain in which the results we can prove in that domain can be represented as consequences of a deductive axiomatic system?

What happens if a scientific theory is successful, but not in the sense that the equations of the theory allow it to be represented [explicitly] as a deductively complete and categorical system?

(ask me about '[explicitly]' during discussion, if you're curious)

the human condition (or, life in the big theoretical city):

for every theory we have, we know almost nothing about the nature of “generic solutions” to the equations of motion or field equations—individual models that we have no idea how to construct in any way graspable by the human mind, or indeed even how to identify if someone gave us one gift-wrapped

(if one holds that a theory is characterized in a strong sense by its family of “models”, then the overwhelming majority of the content of a theory is something that we do not know, are in fact nowhere near knowing, and have good reason to think we will never know in anything resembling thoroughness and detail—epistemologically speaking, to postulate the very idea of a theory as a well defined structure is an act of faith)

what to do, then, when you can't solve equations, and:

1. either other mathematical difficulties prohibit approximative methods and numerical simulation (as in classical general relativity and semi-classical gravity);
2. or one does not want detailed information about individual models or individual possible solutions (because, *e.g.*, the only ones we can derive are trivial in one way or another—as in GR and SCG),
3. and narrow classes of solutions (depending on exact symmetries, *etc.*) just won't do the job?

especially when it is difficult to experimentally access regimes in which one expects the behavior of interest to manifest itself

Then one can try to prove general results; but what exactly is it that one is then doing?

either assume something of a suitably generic character (e.g., an energy condition), and try to derive something of a suitably generic character

1. non-existence claims (non-singular cosmological models)
2. rigidity/stability claims (topology—past-singular closed cosmological models satisfying the strong energy condition)
3. scarcity/genericity claims (measure—cosmological models with a Killing field)
4. non-constructive existence claims (classic singularity theorems)
5. non-constructive behavioral claims (formation of closed trapped surfaces under gravitational collapse)

or construct models based on general principles—*schematic models*:

1. fix general principles and generic conditions (the framework of SCG)
2. characterize general structures that embody the principles and conform to the conditions (a quantum field satisfying the semi-classical Einstein field equation on an asymptotically flat black hole spacetime)
3. derive a statement about generic features of the character and behavior of the general structures (Hawking effect: black holes evaporate)

what exactly is it that one is doing with a schematic model?

- ▶ these models often involve behavioral claims (in the sense of a general result)
- ▶ they are not individual solutions to equations of motion or field equations
- ▶ usually, no exact individual solutions are known that represent such systems in the generality postulated
- ▶ they do not otherwise represent individual systems
- ▶ sometimes, they are not grounded in exact, rigorous general results
- ▶ they are not approximations to or idealizations of solutions
- ▶ they rather represent general features that we expect such solutions to have

black hole evaporation by the Hawking process in SCG is a beautiful example

epistemic value:

1. what can we learn? depends of course on the character and the epistemic warrant of the general principles or conditions one has assumed in the construction of the model
2. what can we do with them?

1. general behavioral claims that cannot easily be captured by theorems (Cosmic Censorship: “the maximal globally hyperbolic future developments of generic, physically reasonable initial data are future inextendible to regular Lorentzian manifolds”)
2. or that are such that the theorems that do imply or characterize them are technically abstruse and have no perspicuous physical interpretation, and so the general behavioral claims (“radiating black holes evaporate in a weakly predictable way, but not retrodictably”) are epistemically more valuable for the purposes of further exploratory investigation than the technically secure results (“Let $(\mathcal{M}, g_{ab})$ be a time orientable spacetime and let Σ_1 and Σ_2 in it be closed achronal sets such that Σ_1 is connected and Σ_2 is edgeless. Suppose there exists a point $p \in D^+(\Sigma_1)$ such that $p \notin J^-(\Sigma_2) \cup J^+(\Sigma_1)$. Suppose further that the set K , defined by $K := \Sigma_1 - D^-(\Sigma_2) \cap \Sigma_1$, has the property that $J^+(K) \cap \Sigma_2$ has compact closure. Then Σ_2 cannot be contained in $D^+(\Sigma_1)$.”)
3. especially when the general behavioral claims go beyond “strict interpretations” of the technical results, (e.g., one has to make further assumptions, not rigorously formalizable in our current state of knowledge, about the character of the propagation of quantum fields in a curved spacetime)

- ▶ schematic models thus seem to give us *understanding* in some sense—but what sense is that?
- ▶ what can we *not* learn?

- ▶ SCG is a fascinating case: general results and schematic models seem to be the only way we have to proceed, given the manifest technical difficulty of, and deep conceptual and physical problems involved in, trying to combine GR and QFT, especially with regard to the divergences and other pathologies accruing to solutions of the SCEFE
- ▶ in this case, in the absence of experimental probing, what we cannot learn is whether there is any unimpeachable veracity to the results and models
- ▶ but we can still have some (weak) confidence that we are on the right track, given our epistemic attachment to the general principles (local validity of QFT in curved spacetimes) and to the reasonability of the generic conditions (no causal pathology) used in constructing the schematic models, and our epistemic faith in the plausibility of how we combine them (the SCEFE)

“Understanding” in this sense, then, is something like “the capacity to operate successfully in the scientific enterprise, in all its aspects and parts”, to use our representations and instruments as the basis for the fruitful continuation of the enterprise:

- ▶ as part of evidential warrant in testing,
- ▶ as basis for predictions and characterizations,
- ▶ as inspiration for potentially fruitful new investigations,
- ▶ as the grounds for conceptual clarification and innovation in foundational work;
- ▶ and perhaps most of all to grasp how our representations and our instruments relate to, inform, and substantively contribute to the constitution of each other,
- ▶ and to grasp that in such a way that it founds the work successfully continuing the enterprise.

insight into epistemology and general structure of scientific knowledge (in physics at least)? of its semantics?

1. the nature of the “representation” of physical systems by schematic models is not straightforward—“black hole”, for instance, is not a univocal concept in GR
2. difficult to see, e.g., how to make sense of what these can teach us using something like the semantic view:
 - ▶ the whole point is we don’t have access to the models—so, *inter alia*, we cannot characterize their empirical content or judge their empirical adequacy using any of the tools it provides
 - ▶ rigidity/stability claims and their ilk, in particular, are relations *among* models, which the semantic view is ill-equipped to handle (the syntactic view may fare better here)
3. the ineliminable role of pragmatic factors: often, if not always, the general state of the entire epistemic community bears on whether one is justified in accepting a schematic model and using it as the ground for further argument, explanation or prediction in a way that one has grounds for believing has at least *prima facie* epistemic warrant—what widely accepted good standards of argumentation with regard to a particular theory, for example, constrain what can legitimately be thought of as a “prediction” of the theory? (*n.b.*: these can and do change over time)

1. whatever understanding it gives us, it is not understanding that can be used as evidential warrant for the theory in any straightforward, traditional sense, for we cannot test the correctness of the general propositions, only corroborate them in special cases, and often we cannot even do that (the singularity theorems of GR)—not the problem of induction, but rather that we don't know with any exactitude what “the general case” is
2. there is no sense in which it is fruitful to conceive of these as “fictions”, whether felicitous or not
3. but neither is there any clear sense in which they may be (or not) “approximately true”
 - ▶ what is the quantifiable measure of approximation for general behavioral claims?
 - ▶ finding one or a dozen or a million systems that, in some sense, behave approximately like that captured by the schematic model does not show that the schematic model captures something “approximately true” *about the world*, only about the application of the general principles and generic conditions to those kinds of systems

valediction in the spirit of Kierkegaard and Stein, following Patton's pregnant remarks:

Realism is not a proposition whose truth-value can be affirmed. It is a way of thinking, an attitude, a style of moving forward in one's cognitive work, shaping and informing but not contributing to the content of one's thought. It is a leap of the absurd into faith, but at the same time a sober reflection that is only one step in the ongoing dialectical dance of one's cognitive capacities in their attempt to explore the world, itself to be given over to instrumentalism—also a leap of the absurd—as the times demand. And that will give over in its turn back to realism, in the Bacchanalic comedy of thought, which, from time to time, settles down into the stately progression of the tragedy of understanding, and back again and again—eventuating in the pragmatic attitude. And so: one is a realist, and one regrets it; one is an instrumentalist, and one regrets it; one is a pragmatist and learns to take joy in passing from one regret to the other.