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KANT ON SPACE, EMPIRICAL REALISM AND THE FOUNDATIONS OF GEOMETRY*

I. SPACE: KANT'S ANSWER TO BERKELEY

1. *Kant and Berkeley*

One of the first reviews (Garve-Feder, 1782) of the *Critique of Pure Reason* described Kant's system as a form of idealism of a piece with that of Berkeley. Kant (Letter to Garve, August 7, 1783) was not pleased with this comparison. In the Prolegomena (13) he explained that his system, far from agreeing with Berkeley, was the proper antidote to Berkeley's objectionable form of idealism. In an explicit response to the offending review (Prolegomena Appendix) Kant claimed that when Berkeley made space a mere empirical representation he reduced all experience to sheer illusion. Kant continued to stress Berkeley's failure to do justice to the special role of space as source of *a priori* constraints on experience when he distinguished his view from Berkeley's in the second edition of the Critique (B 69–72, B 274, Note on B *xi* of Preface). In spite of these protests, quite a number of subsequent writers have offered interpretations of transcendental idealism that would have Kant in basic agreement with Berkeley. Perhaps the most clearly stated example is to be found in Colin Turbayne's classic paper (1955), but any interpretation that construes the manifold in intuitions as sensations or appearances as subjective contents of experience will make Kant's position true to the spirit of Berkeley's point of view. I shall use Turbayne as an example; but, if the interpretation I propose is correct then the way Kant uses space to support his empirical realism makes his position quite different from Berkeley's or from any kind of phenomenalism or any empiricism based on subjective experiences.

Turbayne claims that Kant's main argument against transcendental realism was anticipated by Berkeley's main argument against materialism. He breaks the argument down into six steps which I paraphrase roughly as follows:

(1) The transcendental realist supposes that external objects of perception have an existence by themselves independently of what we can perceive.

(2) What we can be immediately aware of is only the contents of our own representations.

Carl J. Posy (ed.), Kant's Philosophy of Mathematics, 257–291.

(3) Therefore, it is impossible to understand how we could arrive at knowledge of external objects and we are led to skepticism about their existence.
(4) We are also led to skeptical idealism – the doctrine that we can only know the contents of our own representations.
(5) Skepticism about external objects can be avoided by giving up transcendental realism and adopting transcendental idealism – the doctrine that external objects are appearances and so are contents of representations.
(6) This supports empirical realism – the doctrine that we have immediate perception of external objects.

Something like this kind of argument against transcendental realism does seem to be an important part of Kant's Copernican revolution in philosophy.[1] Turbayne uses quotations from Berkeley and Kant to illustrate their agreement at each step. The last two steps represent the official position which combines transcendental idealism with empirical realism.

Fifth Step

Kant: (Transcendental Idealism). External bodies are mere appearances, and are therefore nothing but a species of my ideas, the objects of which are something only through these ideas. Apart from them they are nothing (A 370. Cf. A 491, *Prolegomena 13*).

Berkeley: As to what is said of the absolute existence of unthinking things without any relation to their being perceived, that seems perfectly unintelligible. Their *esse* is *percipi*, nor is it possible they should have any existence, out of the minds or thinking things which perceive them. (Prin. 3).

Sixth Step

Kant: (Empirical Realism). I leave things as we obtain them by the sense their reality (Proleg, 13). In order to arrive at the reality of outer objects, I have just as little need to resort to inference as I have in regard to the reality of the object of my inner sense... For in both cases alike the objects are nothing but ideas, the immediate perception of which is at the same time a sufficient proof of their reality. (A 371)... An empirical realist allows to matter, as appearance, a reality which does not permit of being inferred, but is immediately perceived. (A 37 N).

Berkeley: I am of the vulgar cast, simple enough to believe my senses and leave things as I find them (Hylas III). I might as well doubt of my own being, as of the being of those things I actually see and feel... Those immediate objects of perception, which according to you, are only appearances of things, I take to be the real things themselves.

If by *material substance* is meant only sensible body, that which is seen and felt... then I am more certain of matter's existence than you, or any other philosopher, pretends to be (Hylas III).

As these quotations show, Berkeley certainly did not describe his position as one which reduces all experience to illusion.[2] He regarded his idealist account of bodies as the proper defense of common sense empirical realism against skepticism. This is exactly the virtue Kant claimed for his own transcendental idealism.

Space is one issue on which Kant and Berkeley clearly differ. Kant held that we have knowledge *a priori* about space while Berkeley held that all spatial concepts are merely empirical. According to Berkeley (Theory of Vision 153–154) even the three-dimensionality of space is something that must be inferred from experience by associating visual with tactual sensations.

As we said above, Kant regarded this rejection of *a priori* constraints on space as a fatal flaw in Berkeley's account of the difference between truth and error. According to Turbayne (p. 236), Berkeley based the distinction between truth and error on the coherence of our ideas with one another in experience and Kant is committed to the same kind of account. He suggests (pp. 243–244) that Kant's appeal to their differences over space was no more than an attempt to keep his readers from realizing that this basic position was essentially the same as that of the infamous Berkeley.[3]

2. *Sellars' objections to phenomenalism*

On Berkeley's version of the coherence account Macbeth's dagger is illusory because he cannot grasp it or cut with it – the sense data involved in his experience do not fit into the sort of coherent pattern with other sense data that constitutes seeing a real dagger. Uniformities among our sense data let us coordinate sight with touch and make a host of specific correlations among our subjective experiences. When these uniformities break down we find that our judgments have been in error. When Macbeth sees the dagger apparition hover in the air, he has some grounds for judging that it is not a real dagger – real daggers don't appear to hover in the air without visible support. Upon attempting to grasp it he would have more evidence – real daggers resist when grasped. On this view such breakdowns among the uniformities that constitute experience of real daggers are what make Macbeth's dagger count as illusory.

In order to accurately reconstruct the common sense distinction between truth and error we must be able to account for cases where one person's experience is ground for judging correctly that another's judgment is in error, even if the other doesn't realize his error. When I am situated so as to see that you are standing in front of an empty facade, as you claim to be in front of a house, I can correctly judge that you are in error even if you don't think so. The uniformities that ground an adequate coherentist account must apply impersonally to the experiences of all of us.

In addition to applying impersonally to different observers the uniformities

that account for the difference betwen truth and error must apply to possible as well as actual experiences. Macbeth's dagger is illusory even if neither he nor anyone alse ever actually tries to grasp it or cut with it. It is illusory because one would not be able to succeed if he were to make the attempt. Even if no one is situated to observe them a real house has other sides and insides. Were one to make the appropriate observations he would have the appropriate experiences. Any idealist who leaves everyday empirical things as they are must believe and have good reason to believe many counterfactuals of this sort.

In his discussion of phenomenalism Wilfrid Sellars (1963, pp. 60–106) argues that such counterfactuals cannot be analysed into uniformities among actual sense contents. He points out that in order to specify the appropriate antecedents for the counterfactuals in question one needs to refer to external objects. One striking example is the need for antecedents such as looking from different perspectives. The counterfactual arrangements of bodies in space that would re-position the observer with respect to the object would, themselves, have to be formulated in terms of counterfactual as well as actual sense contents. Thus the very conditions that would be used to define these possible sense contents would have to be based on other conditionals of the same sort.

According to Sellars (1963, p. 80), a phenomenalist might reply by claiming that there are independent general laws about sense data that do not need to be formulated by reference to external bodies and which can be supported by induction based on actual sense data alone. Sellars' answer is that what the phenomenalist needs are generalizations which would apply impersonally, but the best that the phenomenalist can get are uniformities that are valid only for his own particular experience.[4]

For Kant the fundamental *a priori* constraint that space is three dimensional together with the *a priori* constraints on shape and perspective that can be established by geometrical constructions provide richer material for generating an empirical realism. I shall argue that the species of representation Kant uses to account for external bodies is the kind of objective perception exemplified by observations of those perceptible features presented by a three-dimensional object at a specific location and orientation with respect to the observer. Kant's *a priori* constraints build in the assumption that such an object has another side even if only one side is being observed. They also require that the object has a determinate shape that is systematically related to an indefinitely large array of perspectives from which it could be observed.

These spatial assumptions provide exactly what is needed to get around the objections we have considered. The antecedents of the required conterfactual conditionals can be cashed out as specific alternative arrangements of the object and the observer's body in space. Grasping at the dagger is bringing one's hand into the appropriate location and orientation as one squeezes. Similarly, the antecedents relevant for observing other parts of the house are generated by specifications of locations and orientations for the body of an observer relative to the house.

The various uniformities on shape and perspective that support the specific content of these counterfactuals are impersonal in just the way required. That a quarter-shaped object will present a circular aspect to an observer who looks at it from a perspective orthogonal to and centered on its head's side and present an elliptical aspect to one who looks at it from an appropriately different angle is not something idiosyncratic to any particular observer. Such laws are part of what is to count as normal observation of shaped objects in space.[5]

To the extent that Berkeley's position is vulnerable to these objections to phenomenalism, while Kant's *a priori* constraints on space get around them, it is plausible to argue that Kant can be taken at his word when the claims that Berkeley reduced experience to illusion when he made space a merely empirical representation.[6]

3. *Refutation of idealism*

In addition to the objections we have considered Sellars (1963, pp. 83–84) also argues that a phenomenalist is committed to the external world of bodies in space and time when he refers to perceivers and their personal identities. This is one of Kant's own arguments. It is a major theme in the transcendental deduction and the refutation of idealism. Several other writers, including notably Peter Strawson (1966), have argued that Kant is correct on this point because the path traced out by a person's body as he moves about over time through an enduring world of external bodies in space is all that provides for his ability to collect his various subjective episodes into an experience belonging to a single person.

Margaret Wilson (1972, pp. 597–606) has suggested that the foregoing objection only shows that one must use external body concepts to describe the subjective contents of experience and does not show that judgments about external objects have to be known to be true. She argues that even if this undermines a position which attempts to reduce all experience to sensory

contents alone it need not be decisive against Descartes' more modest skeptical position.

According to Wilson (*ibid.*, p. 603) Descartes' only essential contentions are

> (1) Our most confident ordinary employment of physical object concepts is in a significant sense compatible with the non-existence of physical objects, and
> (2) Judgments which purport only to describe our experience, without claiming the actual existence of entities other than ourselves, are not similarly challengeable.

She uses the following example of Descartes' demon hypothesis in action to support the claim that these contentions are plausible:

> Consider, the Cartesian may say, the case of a man approaching an oasis across the desert. First he perceives only the tops of the palm trees. After a while he perceives the trunks. Although his perceptions of the trunks occur after his perceptions of the leafy tops, he will naturally take both to be perceptions of one set of stable objects, not of temporally successive sets of objects. As he gets nearer, he sees a bird in one of the trees. He sees the bird stretch open its beak, then close it, then fly off. Then he hears a shrill note. While he perceives the bird-flight before the bird-cry, he takes it to have occurred afterwards. In other words, he implicitly makes all the usual distinctions between subjective and objective time order, in complete conformity with the examples of the Second Analogy. Now let us suppose (1) that the oasis was a mirage; or (2) that the man was not awake; or (3) that he was in the clutches of a deceitful demon or super-scientist, who was in some manner providing him with a fantastic series of perceptual experiences. Certainly, the Cartesian will continue, there is a sense in which this man not deceived about the character of his own perceptual experiences. Yet he certainly was deceived in taking them directly to represent an outer reality. Now, how can we *ever* be sure that our 'outer experience' is not deceptive in precisely this manner, etc....?

The science fiction version of the demon hypothesis is especially compelling today. How do I know that I'm not just a brain in a vat. Perhaps my present experiences, and indeed my whole life's experiences, are nothing but responses by my brain to artificial inputs provided by ingenious super scientists. This kind of hypothesis seems to obviously a coherent possibility in principle, even if it cannot be achieved yet by today's scientists, that it has revived the demon hypothesis as an epistemological puzzle of concern to philosophers.[7]

The two contentions Wilson acribes to the Cartesian correspond to the first two steps in the paralogism argument that made Kant look like Berkeley.[8] They have the effect that our judgments about the subjective contents of our experiences are immediate, but that the existence or non-existence of external objects is independent of our judgments about them. The demon hypothesis challenges our ability to arrive at knowledge of external objects (step 3) and invites the skeptical conclusion that all my knowledge is limited to the subjective contents of my own experience.

On Berkeley's version of it, the idealist move in step 5 gives up the Cartesian contention (1), but keeps contention (2). According to Berkeley external bodies are accounted for by visual and tactual ideas that contain nothing beyond what is immediately given to the senses. On his view the *esse* of these ideas is *percipi* so that my judgments about what I perceive immediately are incorrigible.[9] Kant's skeptical idealist (step 4 in the paralogism argument) could say that he gives up (1) and that he interprets each perception claim about external bodies as asserting no more than the subjective content of that particular perception. He could then say, as Berkeley does, that in his view, our perceptions of external objects are immediate; but, such a view would certainly not provide for an empirical realism.[10]

The Berkeley that Turbayne shows us would interpret my judgments about external objects as asserting appropriate uniformities among the subjective contents of my experience.[11] On the assumption that (contrary to what I have argued above) these uniformities can be made available within Berkeley's framework, this more realistic kind of subjective idealism does defuse the skeptical argument. On this idealistic assumption the demon hypothesis is incoherent because the truth of my claim that external bodies exist comes down to the same thing as having my subjective experiences satisfy the appropriate uniformities.

On the interpretation I shall propose, there is an important difference between the ways Kant and Berkeley give up contention (1). According to transcendental idealism external bodies are accounted for by appearances and appearances can be immediately perceived by us. The difference is that the appearance I perceive now is correctly construed as an object the existence of which is independent of my perception of it. On this interpretation, appearances are objective rather than merely subjective contents of perception and my judgments about them are not incorrigible.

Even though appearances are empirically real so that they are independent of anyone's actual perception of them they are not transcendentally real because they are not independent of what could be perceived by observers like us. Kant still has available an idealistic answer to the demon hypothesis. On the interpretation I shall defend I can assume that my judgment about an appearance I perceive is false only by assuming that it fails to cohere with a host of other claims about outer appearances which I assume to be true. There is, on this view, no way to coherently assume that all my judgments about outer objects are false.

By giving this objective account of appearances Kant has broken the connection between immediate perception and incorrigibility. He holds both that my perception of the appearance presented to my senses now is immediate

and that it includes an objective judgment that can in principle be mistaken. On this view the fact that my present judgment is corrigible does not mean that it is in any way doubtful. My perceptions of the outer appearances presented to me are not mediated by any more direct perceptions of the subjective contents of my experience. Even though they are corrigible they are as immediate and certain as any perceptions I can have.

Kant's transcendental idealism gives up contention (2), as well as contention (1). On his view my judgments about the subjective contents of my own experience are no more immediate than my judgments about the outer appearances I am presented with. According to the refutation of idealism (B275–279), my having knowledge of the determinate temporal sequence of my subjective experiences depends upon my having determinate knowledge about outer things. If this is correct, then the need, pointed out by Sellars and Strawson, to appeal to my body's path through an objective world in order to know how my subjective experiences fit together in time requires knowing the truth of some judgments about outer things.[12]

Though Kant and the skeptical idealist agree in treating my judgments about external objects and my judgments about my subjective experience as equally immediate they do so in opposite ways. Where the skeptical idealist would treat claims about external objects as incorrigible claims about subjective experience, Kant would treat my judgments about my subjective experiences as no less corrigible than my judgments about external objects.

Recently Hilary Putnam (1981, pp. 1–20) has argued that the demon hypothesis 'I am a brain in a vat' is self refuting because if it were true I would be unable to use the word 'vat' to refer to actual vats in the world. Putnam (p. 62) points out affinities between his views, Kant's position on sensations, and Wittgenstein's private language argument. Other writers including Sellars (1963, Chapters 3 and 5; 1968, Chapters I and II) and Jonathan Bennett (1966, pp. 202–209) have also given interesting arguments in support of Kant's position that bring out affinities with Wittgenstein.[13] I think that Kant's argument can be profitably interpreted along the lines these writers suggest.

For Kant, just as for Putnam, the demon hypothesis about my own case gives up what is needed in order for me to make some objective reference it requires me to make. Unlike Putnam, Kant focuses on the objective reference required to have knowledge of the temporal order of my own experiences. If my past tense judgments are to connect together in an appropriate way then I must be able to use 'now' to demonstratively refer to a location in an objective time order that defines a past for these judgments to refer to. If my

experiences are construed as supporting nothing beyond incorrigible claims about subjective contents then they cannot support any such objective time order. On the assumption that all my outer experience is hallucination I have given up any grounds on which I could know that the temporal order my experiences seem to have is the one they actually do have. The demon hypothesis is incoherent because on it there is no way to prevent my experience from collapsing into a solipsism of the present moment.[14]

II. KANT'S EMPIRICAL REALISM

1. *Appearance: The undetermined object of an empirical intuition*

The following taxinomy can help to explicate the species of representation[15] that count as intuitions:

> The genus is *representation* [Vorstellung] in general 'repreasentatio). Under it stands the representation with consciousness (*perceptio*). A perception which relates only to the subject as a modification of its state is *sensation* [Empfindung] (*sensatio*),. an objective perception is *cognition* [Erkenntnis] (*cognitio*). This is either *intuition* [Anschauung] or *concept* [Begriff] (*intuitus vel conceptus*). The former relates immediately to the object and is single, the latter relates to it mediately by means of a feature [Merkmal] which several things may have in common. The concept is either an *empirical* or a *pure* concept; and the pure concept, so far as it has its origin only in the understanding (not in the pure image of sensibility), is called a *notion* (Notio). A concept formed from notions and transcending the bounds of experience [Erfahrung] is the idea [Idee] or concept of reason [Vernunftbegriff]. (A 320/B 377).

This passage distinguishes intuitions from sensations on the one hand and concepts on the other. I shall deal with the distinction between intuitions and concepts before dealing with the distinction between intuitions and sensations.

An intuition is single and relates to its object immediately while a concept relates to its object mediately by means of a feature which several things can have in common. An intuition is single in that it is a singular representation – one that can have only one particular object – while a concept can be satisfied by many distinct instances. In this respect, the distinction corresponds roughly to that between an individual referring expression and a predicate expression in symbolic logic. Hintikka has argued (1969) that this sort of logical distinction between particular ideas and general concepts captures the essence of Kant's use of intuition. Other writers (Parsons, 1964; Sellars, 1968; Howell, 1973) have argued that, on Kant's view, a demonstrative element is essential to any intuition. The emphasis, in this passage, on the immediacy with which an intuition is in relation to its object may support

these writers. Our problem is to get clear about the sort of empirical intuitions that Kant uses to account for external bodies. For these intuitions at least, a demonstrative reference to a specified actual instance is essential.

Consider the empirical intuition I have as I observe three coins arranged on the desk before me. Presumably, the object of this intuition is a complex of several individual things. It may be, as Kant sometimes suggests, that any such intuition of a complex is a complex of simpler intuitions; but, even if this were so, a complex intuition would still be an intuition. My intuition can still be singular in that it unambiguously designates this one instance of the arrangement of coins. The important singularity of intuitions, at least of empirical intuitions of the sort that concern us here, is to refer demonstratively to a single instance. It does not matter whether the specified instance turns out to be simple or complex. In the example under consideration, the object of my intuition is whatever is actually present now at the location I specify when I refer to the coins before me on my desk.

Kant distinguishes intuitions from sensations in the following manner: an intuition is a cognition or objective perception while a sensation only relates to the subject as a modification of its state. Since Kant would regard Berkeley's ideas as mere sensations, this distinction between intuitions and sensations is vital to the difference between his transcendental idealism and Berkeley's subjective idealism. Adequate treatment of this difference will require explicating the role of sensation in empirical intuitions. This explication will benefit from a consideration of additional passages in which Kant distinguishes between empirical and pure intuitions.

Among these passages the following paragraph deserves to be quoted in full because this will help set the stage for the explication to follow:

> Our knowledge [Erkenntnis] springs from two fundamental sources of the mind; the first is the receiving of representations (the receptivity for impressions), the second is the power to know [erkennen] an object through these representations (spontaneity for concepts); through the first an object is *given* to us, through the second it is *thought* in relation to that representation (which is a mere determination of the mind). Intuition and concepts, therefore, constitute the elements of all our knowledge [Erkenntnis], so that neither concepts without an intuition in some way corresponding to them, nor intuition without concepts, can yield knowledge [Erkenntnis]. Both may be either pure or empirical. They are empirical when they contain sensation (which presupposes the actual presence of the object), and when there is no admixture of sensation with the representation they are pure. Sensation may be called the material of sensible knowledge. Pure intuition, therefore, contains only the form under which something is intuited, and pure concept only the form of the thought of an object in general. Only pure intuitions or concepts are possible *a priori*, empirical ones must be *a posteriori* (A 50/B 74–A 51/B 75).

According to this passage, an intuition is empirical when it contains sensa-

tion; moreover, sensation presupposes the actual presence of the object, and it is through receptivity for impressions that objects are given to us. The passage also implies that all my information which is not contributed by my own mental apparatus must come from the input of sensations to that apparatus.

That there is a tight correspondence between such characteristics as the shape and hardness of a physical object, e.g. a rubber ball, and the kind of experience I have when I look at and handle it, is one of the familiar facts of life. When I look at the ball, what I see depends on the ball, the perceptual circumstances, and my psychological circumstances; but, given the specification of these contingencies it is independent of my decision.[16] I can decide to look or not to look but, if I look, what I see is not all up to me.

According to Kant, sensations are essential to this independence. In this system they link up my mental machinery with the world:

> The effect [Wirkung] of an object on the faculty of representation, so far as we are affected by it, is *sensation*. That intuition which is in relation to the object through sensation is called *empirical*. (A 19–20/B 34)

Notice that Kant does not say that sensation is perception of what it corresponds to. On the contrary, in all three passages quoted above, he carefully restricts sensation to a modification of the state of the subject only, to that representation which is a mere determination of the mind, and to the effect of an object on the faculty of represenation *so far as we are affected by it*. The sort of perception which is in relation to an object, through sensation, is empirical *intuition* – not sensation itself.

How does an empirical intuition contain sensation? Consider the following proposal: Just as an instance of a sign design can function as a token for a sentence in so far as it is subjected to rules that govern correct usage for that sentence, so also may an array of sensations function as the token for an empirical intuition.[17] The connection between any token and the represenation it is used to token is provided by the rules that govern correct tokening of the type of representation in question. An empirical intuition is just a sensation episode that is subjected to rules appropriate for tokening that specific type of intuition.

According to this proposal, seeing that there are coins on my desk is distinguished from merely having sensations of a certain kind in that seeing is subject to rules that govern judgments about objects of experience while mere sensations are not. This makes the distinction between an empirical intuition and mere sensation analogous to the distinction between asserting that there

are coins over there and merely mouthing the words. On the other hand, also according to this proposal, *seeing* that there are coins over there is distinguished from merely asserting the corresponding judgment in that seeing required being in perceptual circumstances appropriate to produce the corresponding sensations while asserting does not require any such immediate relation to the object of the judgment.

The last passage quoted, where sensations and intuitions were characterized, continues with the following sentence, which characterizes appearance as the undetermined object of an empirical intuition:

> The undetermind object of an empirical intuition is called appearance (A 20/B 34)

In so far as it is an object of an outer intuition an appearance must be subject to the general rules that characterize the pure concept of an object in space. These rules are generated by the role of space as the pure form of all outer intuition and require that the object have a location with respect to the observer in three-dimensional space as well as satisfy all the constraints on shape and perspective that can be established by geometrical constructions. The appearances Kant uses to account for external bodies are objects of outer intuitions; therefore, whatever may be undetermined about them, they must at least satisfy these general rules.

I propose that the object of an outer empirical intuition is undetermined in so far as it is subjected to none but these general rules for objects in space. Consider again the empirical intuition I have as I observe the coins on my desk. I leave the object undetermined when I limit my judgment about it to just those perceptible features actually presented to me now, together with whatever these features imply according to the general rules governing objects in space. The appearance is simply something – *qua* presenting the aspect of a specific triangular array of three dime-shaped objects viewed from my relative location and perspective. The basic idea here is that the features which generate the content of an appearance are just those perceptible features that are actually exposed to the appropriate senses of the observer.[18] The shapes on occluded sides of the coins are not part of the content of this appearance. The very same appearance could have been presented by rods embedded in the desk with exposed ends shaped like tops and edges of dimes; it could also have been presented by an appropriately focused hologram. An hallucination would not count as observing the same appearance even though I might mistake one for such an observation.

When I judge that what I see are coins, I subject the object of my intuition to rules that require more of it than just this appearance. I require, among

other things, that coins be rigid enough to resist when touched and that they have appropriate boundaries on the sides not being observed. The additional content provided by the empirical concept of a coin connects the appearance presented to me now with other appearances that are not now presented to me but would be presented to an appropriately located observer. Were I to pick up and examine one of the coins I would be presented with an appearance that included tactual as well as visual inforamtion. All of the directly presented features, the shape, texture and resistance presented to my fingers as well as the shape presented to my sight are located in one space relative to the location and orientation of my body.[19] As I construe them here, Kant's appearances are just those objective properties of actual things in space that follow geometrically from those perceptible features that would be presented directly to the senses of an appropriately situated human observer.

In many passages Kant tells us that imagination is the process by which sensations are worked up into empirical intuitions. According to this picture we can think of an outer appearance as that set of sensible features which I intuit in an object simply through the taking up of sensations into the imagination according to the general rules for having an outer intuition at all. The appearance is the content of a minimally conceptualized intuition. The object of such an intuition is characterized as this something – *qua* having the perceptible features generated by the imagination under the guidance of only my present sensations and the pure concept of a spatial object.

Even when the ascription of content to the object of my outer intuition is limited in this way, the general spatial rules require some definite connections between the appearance actually presented to me and further appearances that would be presented to appropriately located observers. For example, (as we remarked above, Note 5), the shape presented to me is systematically related to what would appear to other perspectives. This commitment to further appearances makes the ascription of even these most directly perceptible features both objective and corrigible. I take this to be Kant's main point when he insists that empirical intuitions are objective perceptions and not mere sensations.

2. *Transcendental idealism and empirical truth*

Kant's most developed exposition of the way his transcendental idealism supports an empirical realism is to be found in the long paragraph (A190–191; B235–236) which opened his first edition version of the second analogy and was retained unchanged as the third paragraph in his second edition version. I

shall attempt to show how my explication of Kant's basic conception of appearance as the undetermined object of an empirical intuition illuminates what I take to be the two central ideas Kant introduces in this celebrated passage.[20] One of these is a transcendental sense of 'appearance' according to which even such a complex solid object as a house counts as an appearance. The other is an account of empirical truth that is objective and yet avoids the demon argument which plagues the transcendental realist conception of truth as correspondence with things as they are in themselves.

Kant tell us that a house is not a thing in itself, but an appearance. He explicates this by glossing 'appearance' as "a representation the transcendental object of which is unknown". He then asks what we are to understand by the connection of the manifold in the appearance itself, when an appearance is neverthelsss not anything in itself.

> Now, as soon as I unfold the transcendental meaning of my concepts of an object, I realize that the house is not a thing in itself but only an appearance, that is, a representation, the transcendental object of which is unknown; therefore, what am I to understand by the question: how the manifold may be connected in the appearance itself (which is yet nothing in itself)?

When I judge that what I see before me is a house, I ascribe more to the object of my experience than just those directly perceptible features that are now afforded to my senses. These additional ascriptions go far beyond what the general concept of an object in space requires in order that the shape from other perspectives cohere geometrically with what I observe. Accordingly the house before me is not as undetermined an object of empirical intuition as the perspective-bound appearances I have been explicating. Lewis Beck (1978, pp. 143, 146) is surely correct that Kant uses a thicker notion of appearance when he applies it to such complex objects as houses.[21] He calls this Kant's *transcendental* sense of 'appearance' and identifies the more perspective relative notion I have been explicating with what he takes to be Kant's contrasting *empirical* sense of 'appearance'.[22]

Kant's rhetorical question, at the end of this passage, can be understood as asking how the manifold of a transcendental appearance can be independent even though it contains nothing beyong contents of representations. Kant's somewhat enigmatic answer is given in the next passage, which immediately follows his question in the text.

> That which lies in the successive apprehension is here viewed as representation, while the appearance which is given to me, notwithstanding that it is nothing but the sum of these representations, is viewed as their object; and my concept, which I derive from the representations of apprehension, has to agree with it.

That which lies in the successive apprehension is presumably a manifold of empirical appearances. These appearances have a two-sided character. On the one hand each is the content of an empirical intuition and therefore can be viewed as a representation. On the other hand, as actual features of objects in space, they are connected with one another in a manner that is independent of anyone's apprehension of them. The empirical intuitions in my apprehension make *demonstrative* reference to a spatio-temporal vicinity. The independence of the object of my experience is provided by all the additional empirical appearances to be found in that vicinity. In this way a transcendental appearance that contains nothing beyond contents of representations can, nevertheless, be viewed as the independent object that my concept has to agree with.

My concept is derived from the representations of apprehension, in that the perceptible features actually presented to me lead me to judge that what is before me is a house, rather than (say) a ship, tree, or an empty stage prop. If my judgment is correct, this concept has to agree with whatever turns out to be the actual object of my experience.

This demonstrative reference, rigidly denoting whatever is at a spatio-temporal vicinity, is the most important contribution of Kant'*a priori* requirement that the object of an outer empirical intuition have a determine location relative to the body of the observer in three-dimensional space. The specific geometrical constraints on the relation of three-dimensional shape to perspective also play an important role. They provide a framework which allows the identification of the house as an independent empirical object which underlies all the appearances in its manifold. This object is whatever affords the mereological sum of all the three-dimensional shaped surfaces revealed in these various empirical appearances. It is the empirical substance of which the various perceptible features revealed in these appearances are determinations.

On this account when I look at a quarter from a perspective 45° from perpendicular to its head side, I directly see its non-occluded surface as something shaped like an appropriate part of a three-dimensional disk located and oriented in the way specified. There is none of that difficult business of seeing it as elliptical but judging it to be round which plagued G. E. Moore. I think that allowing for direct perception of oriented shaped surfaces in three-dimensional space is fundamental to any Kantian account of how observers from different perspectives can see over-lapping parts of the same empirical substance.

The following account of empirical truth completes Kant's explication of how to construe appearance as the formal-being referred to by the representations in my apprehension:

> One soon sees that though truth is agreement of the cognition [Erkenntnis] with the object, only the formal conditions of empirical truth can be in question here, and appearance in contrast with the representations of apprehension can be represented as an object distinct from them only if it [the appearance] stands under a rule, which distinguishes it [the apprehension of this appearance] from every other apprehension and makes necessary some one particular kind if connection of the manifold. That in the appearance which contains the condition of this necessary rule of apprehension is the object (A 191/B 236).[23]

Consider my observation of my own house as I stand before it. Let *A* be the proposition that what is before me now is that particular house. If it is empirically true that *A* obtains here-now, then the object of my experience must agree with my cognition – i. e., with my judgment that *A* is the case here now – and must also be representable as something independent of the representations in my apprehension of it. Therefore, whatever is before me over there must satisfy the condition of a rule that distinguishes it from any possible object of experience that fails to be an instance of *A*. This rule distinguishes my apprehension, *qua* an apprehension of an instance of *A*, from any other apprehension – i. e., from any apprehension of anything that fails to be an instance of *A*.

In this example the representations of my apprehension are my perceptions of the perceptible features actually presented to me. These would include the shape relative position and orientation of the facing surfaces, etc. The object of my experience is whatever is present at the appropriate, spatio-temporal vicinity of the location to which I now refer demonstratively. There are many more perceptible features there to be observed than the ones now presented to me. It is this demonstrative reference to an inexhaustibly rich source of additional perceptible features that gives the object of my experience its independence from the representations in my apprehension of it.

I believe that this account of empirical truth is the heart of Kant's Copernican revolution in epistemology. In place of the transcendent notion of truth as correspondence with the way things really are in themselves, he gives us empirical truth as correspondence with what can count for us as the actual objects of our experience. This transcendental idealism avoids the Cartesian argument for skepticism at least as well as Berkeley's subjective idealism. The demon hypothesis cannot be empirically true because it assumes away the demonstrative reference to an independent object of experience required to provide the empirical content that could make it true. The advantage over Berkeley is that it provides for the independence and objectivity required by our common sense empirical distinctions between truth and error.

III. PHAENOMENA: APPEARANCES THOUGHT ACCORDING TO THE UNITY OF THE CATEGORIES

1. *The principle of extensive magnitudes*

So far as we have explicated it, Kant's empirical realism supports the common sense realm of candlesticks, ships and houses against skeptical reductions to subjective contents of experience; but, our explication has been limited to observables in a sense close to that advocated by van Fraassen (1980) in his anti-realist constructive empiricism. It would be disappointing for some of those who see Kant as providing a foundation for scientific methodology to find that his empirical realism does not support existence claims about the non-observables postulated by modern science.

In a passage at (A 249) Kant tells us that:

> Appearances, so far as they are thought according to the unity of the categories, are called Phaenomena.

The various categorical principles Kant argues for impose additional constraints on the basic idea of apprearance as the undetermined object of an empirical intuition. I think that these constraints transform the account of empirical truth by adding commitments that go beyond observables. I shall illustrate this point by considering some consequences of the axioms of intuition.

According to the *Axioms of Intuition* all appearances are extensive magnitudes. When Kant tells us that these magnitudes are determinate he is requiring that, for example, at any instant in time ratios of lengths along any specified dimensions of an object in space determine specific real numbers. It is important to note that Kant takes this commitment to determinate extensive magnitudes as a constitutive condition on appearances. When explicating outer-appearance as the undetermined object of an empirical intuition I claimed that the content of such apprearances is limited to features to objects in space that are directly accessible to the senses of observers like us together with whatever these features imply according to the rules constituting the general concept of an object in space. We have seen that the qualitative geometrical constraints on perspective require that judgments about shapes directly presented to an observer at one perspective carry systematic commitments to further aspects that would be presented to observers at other appropriately oriented perspectives. Now we see that the general concept of an object in space also carries commitment to determinate extensive magnitudes. Even when I limit my judgment about the object of my empirical intuition to

the outer appearance presented to me I must include in the content of my judgment that, at any given instance, e. g. now, and relative to any appropriate specification of a standard, the length along each specifiable dimension of the object has a determinate value.

Consider a spatial example where lengths are being compared. Let $A_1 \ldots A_n$ be some finite sequence of propositions such that each A_i asserts that the distance (relative to a specific meter stick) between the centers of mass of two quarters on my desk now falls in the i th of n adjoining tiny length segments. We are to make these segments small enough so that each A_i is below the threshold of human sensory detection, but large enough so that the disjunction of all the A_i's is something we can observe to be true. This holds if we limit our observations to whatever comparisons of length we can establish with unaided sight and touch as we lay the meter stick across the coins, and it continues to hold even if we allow what we observe to be enhanced by the best measuring instruments science can provide. The principle of extensive magnitude makes commitments that go beyond the resolving powers of human observation even if these powers are extended by instruments.

An appearance includes the specification that each spatial dimension in it be a determinate extensive magnitude. But, even given the specification of an appropriate standard and time, does it also include specification of what the exact value of each of these magnitudes is? On the account I have been proposing the answer to this question is no, because these exact values are not implied by observable features even under the most lavish construal and application of mathematical rules constituting the general concept of an object in space. This has the effect that the disjunction $A_1 \vee \ldots \vee A_n$ will be empirically true even though none of its disjuncts is. Similar examples will show that an existential statement can be empirically true even when each of its instances is empirically indeterminate. Indeed, it will be empirically true that each magnitude has some determinate value, even though for each magnitude it will be empirically indeterminate exactly what this value is.

Another option would be to include the specification of the exact value of each magnitude as part of the empirical content itself. This would remove any empirical truth value gaps generated by the commitment to determinate extensive magnitudes, but it would lead to a problem pointed out by Charles Parsons (1964). On this option Kant is faced with a dilemma. Either he would have to claim that humans have the ability to, in principle, make infinitely fine discriminations of extensive magnitudes, or he would have to claim that what counts as empirical content is not determined by the discriminations that humans, even in principle, could make. Neither horn of this dilemma is very

attractive. The first horn would seem to commit him to something patently false. Certainly our best instruments now fall far short of the precision required, and, though we can expect improvements that allow closer approximations. nothing in the way such improvements have been made in the past makes it plausible to suggest that such approximations could even in principle culminate in exact values. The second horn of the dilemma, on the other hand, would seem to fly in the face of the whole idea of Kant's Copernican revolution in epistemology. If empirical content is inaccessible to our senses, even augmented by the best instruments we could in principle, devise, then how could we know empirical truth from error?

On the option I propose this dilemma is avoided in a way that seems in keeping with Kant's Copernican revolution and with his specific account of the a priori as something we impose on nature. The principle of extensive magnitudes shows that the basic account of empirical truth carries commitment to a more deterministic ideal in which the value of each magnitude is exactly settled. Any coherent way of filling out the specifications required by this ideal that is left open by what is settled by the empirical content will act as an admissible valuation in a supervaluation semantics appropriate to the account of empirical truth.[24] Whatever holds according to every way of filling out the ideal by arbitrarily assigning these values in some coherent way will count as empirically true. Therefore, the principle of extensive magnitudes contributes considerable strength to the account of empirical truth, even if we allow that the exact values of these magnitudes are empirically indeterminate. This way that empirical truth carries commitment to a more determinate mathematical ideal turns it from a fairly restrictive observationalism to a possible foundation for scientific realism, without violating the spirit of Kant's Copernican revolution in Epistemology.

2. *The principle of the first analogy*

Though I shall not argue the point here, I think that Kant's arguments for the principles of the analogies can best be understood as an attempt to show that extension of the empirical content that can be appealed to in the account of empirical truth beyond what is presented to the observer here-now to additional observables at other times and places requires commitment to enduring substances and causal laws. If, as seems to be the case, the only candidate for the enduring substances that underly many observable changes are non-observables, then the first analogy, as well as the axioms of intuition, will carry commitment to the empirical reality of some non-observable entities.

Kant tells us (B 233) that the principle of the First Analogy can be expressed by the requirement that all change is alteration – a succession of opposite determinations of a substance which abides. This principle requires that any change which we might be inclined to describe as the destruction of an empirical thing must be a succession of opposite states of some underlying substance which persists through the change. Consider one of those familiar white styrofoam coffee cups. Now destroy it by smashing it to pieces. If both the before and the after states have to be determinations of the same substance then that substance cannot be the cup which was destroyed. What is available to both persist through and be relevant enough to ground this change? One obvious candidate is the mereological sum of all the little styrofoam particles. If the after state is these particles all jumbled about in disarray while the before state was these same particles assembled together into the cup, the change can be a proper alteration. Now burn the little pieces of styrofoam. What is available to count as a substance which underlies this change? Presumably, some postulated collection of non-observable entities – something like molecules or atoms.[25] Thus, the principle of the First Analogy, together with such familiar happenings as destructions by burning which break something up into parts smaller than we can observe, seems to carry commitment to just the sort of non-observable theoretical entities dear to the heart of a scientific realist.

3. *Indeterminacies*

We have seen how the principle of extensive magnitudes makes commitments that generate empirical truth value gaps. The same would hold for commitments generated by the First Analogy. Even if some version of the kinetic-molecular theory of gasses turned out to be empirically true of some specified volume of gas there would not be any specific assignment of positions and momenta to the individual particles (at any given time) that would be singled out as the unique empirically true one. Only the existential proposition that there was some such distribution of momenta would be empirically true. There would be a very large range of possible assignments of these magnitudes that would be equally compatible with the apprearances – roughly all ones which afford average kinetic energy values within the observable tolerances.

This possibility of truth value gaps is a feature which my account of empirical truth shares with Carl Posy's (1983, 1984) intuitionistic rendering of Kant's transcendental idealism. I think any account of Kant's position which

takes seriously his relativization of empirical truth to possible objects of experience will have to allow for such indeterminacies. Posy's intuitionistically motivated account is one way to do this. My account of empirical truth with its supervaluation way of dealing with indeterminacies is another. Posy (1983) has done an admirable job showing how his proposal can illuminate Kant's difficult discussion in the First Antinomy. I think the supervaluation approach can offer a comparably illuminating analysis of this difficult passage, but the details will have to wait for another occasion.

IV. GEOMETRY

1. *Kant's commitment to a priori constraints on space*

We have noted (Section I) that, according to Kant, Berkeley reduced all experience to sheer illusion when he made space a merely empirical representation. This suggests that Kant's commitment to the claim that geometry provides knowledge *a priori* of constraints on objects of outer sense is deeper than his desire to provide a philosophy of mathematics. It suggests that he thought these *a priori* spatial constraints are what prevent his apprearances from collapsing into merely subjective contents of experience – that they are what separate his empirical realism from the objectionable form of idealism he attributed to Berkeley.

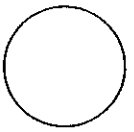
My account of Kant's empirical realism appeals to the constraint that any object of an outer intuition must have a determinate location and orientation relative to the body of the observer, and to the numerous specific constraints on shape and perspective that can be revealed in geometrical constructions. These spatial constraints provide a framework within which appearances can be construed as objective features of things our senses can carry immediate information about. If this account is correct then Kant's defence of empirical realism is based on these constraints on space. So, Kant may have been justified if he thought that his defence of empirical realism would be threatened unless he could appeal to such knowledge *a priori* about space.[26]

The sort of constraints on shape and perspective that my account of Kant's empirical realism needs can be illustrated by Shimon Ullman's (1979) various structure from motion theorems. Ullman's basic theorem concerns orthographic projections.[27]

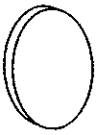
Given three distinct *orthographic* projections of four non-coplaner points in a *rigid* configuration the structure and motion compatible with these views is uniquely determined (up to a reflection about the image plane).

Ullman (pp. 134–6) discusses a neat experiment in which visual information processing supportable by this theorem can be observed. Moving points are projected onto a screen with motions compatible with orthographic projections of points on the surfaces of two rotating transparent coaxial cylinders. As you look you cannot help but see them as points on the rotating rigid three dimensional cylinders. This suggests that we visualize as though we operated with a wired-in program which first looks for some possible rigid body in relative motion interpretation of the sensory input. We can see Ullman's theorem as providing constraints on what can count as a rigid body in relative motion interpretation.

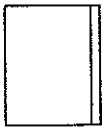
Ullman's shape from motion theorem tells us that a rigid configuration that afforded this orthographic projection



to one perspective, and this orthographic projection



to a perspective corresponding to a 45° rotation to the right could not afford this orthographic projection



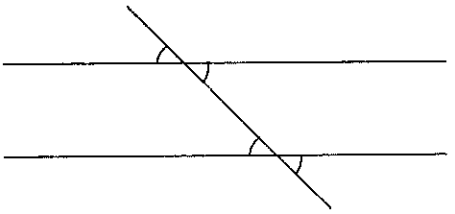
to a perspective corresponding to the opposite 45° rotation. I shall call this my *salient illustration*.

According to Kant such constraints on shape and perspective are built into the structure of space which is the pure form of outer intuition. He holds that geometrical constructions offer us *a priori* knowledge of these structural con-

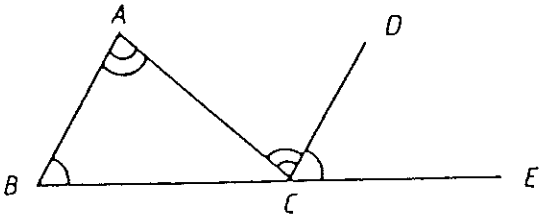
straints that apply to anything that can be an outer object of experience for us. On this view geometrical constructions provide us with a way of making the pure form of our outer intuition transparent to ourselves.

2. *Kant's account of geometrical constructions*

Perhaps the most salient example of geometrical construction in Kant's writing (A 716–17; B 744–5) is the one used in Euclid's proof of Proposition 32 (in the *Elements*) – that the sum of the interior angles of a plane triangle equals a straight angle (180°). Euclid's proof of this proposition appeals to proposition 29 about various equal angles made when a straight line falls on two parallel lines. The following diagram is a construction which shows that all the marked angles must be equal.



I think that, according to Kant, anyone who properly understands this diagram cannot help but be compelled to see that all the marked angles must be equal – and that this would hold for any straight line falling on two parallel lines. The heart of Euclid's proof of proposition 32 is the following construction.



From proposition 29, or by inspection of the present diagram, we see that angle *ABC* equals angle *DCE* and angle *BAC* equals angle *ACD*. From the way we constructed auxiliary lines *CE* and *CD* we can now see immediately that *ACB* plus *ACD* plus *DCE* equals *BCE*, since these three angles together just are the straight angle *BCE*. I think this proof shows the intuitive force that geometrical constructions provide. It is very hard to reason through this diagram without feeling compelled to accept Euclid's general proposition that the sum of the interior angles of a plane triangle equals a straight angle (180°).

Kant attempts to justify this compulsion by his account of geometrical constructions. The following passage gives Kant's explanation of the role of the triangle diagram as a constructive definition of the concept of a plane-triangle.

> To construct a concept means to exhibit *a priori* the intuition which corresponds to the concept. For the construction of a concept we therefore need a *non-empirical* intuition. The latter must, as intuition, be a *single* object, and yet none the less, as the construction of a concept (a universal representation), it must in its representation express universal validity for all possible intuitions which fall under the same concept. Thus I construct a triangle by representing the object which corresponds to this concept either by imagination alone, in pure intuition, or in accordance therewith also on paper, in empirical intuition – in both cases completely *a priori*, without having borrowed the pattern from any experience. The single figure which we draw is empirical, and yet it serves to express the concept, without impairing its universality.

The single empirical figure I draw functions as the pure intuition which underwrites a *real definition* of the geometrical concept of a plane triangle. As a real definition it displays sure marks by which to identify any figure that is to count as a plane triangle and it also provides an actual instance which shows that this concept is not empty. (Parsons, '1969; Beck, 1956)[28]

Kant goes on to tell us more about how it is that this single empirical figure can serve to express a pure geometrical concept without impairing the generality of that concept.

> For in this empirical intuition we consider only the act whereby we construct the concept, and abstract from the many determinations (for instance, the magnitude of the sides and of the angles), which are quite indifferent, as not altering the concept 'triangle'. (A 714/B 742)

These remarks can be usefully amplified by the following passage from the schematism.

> No image could ever by adequate to the concept of a triangle in general. It would never attain that universality of the concept which renders it valid of all triangles, whether right-angled, obtuse-angled, or acute-angled; it would always be limited to a part only of this sphere. The

schema of the triangle can exist nowhere but in thought. It is a rule of synthesis of the imagination, in respect to pure figures in space. (A 141/B 180)

The diagram can express the pure intuition of the geometrical concept of a plane triangle in so far as my reasoning about it appeals only to the schema of this concept. The schema is a rule for the synthesis of imagination required to construct any ostensive representation of a plane triangle.

The following passage contrasts what Kant calls the ostensive character of this sort of geometrical construction with the symbolic constructions to be found in algebra.

thus in algebra by means of symbolic construction, just as in geometry by means of an ostensive construction (the geometrical construction of the objects themselves), we succeed in arriving at results which discursive knowledge could never have reached by means of mere concepts. (A 717/B 745)

It also illustrates Kant's commitment to the claim that Euclid's proposition 32 does not follow analytically from the mere concept of a plane triangle. The result depends essentially on the additional content provided by the construction. The schema for this pure concept is embedded in and shows us constraints on our framework for ostensively recognizing figures in space. Thus, this pure intuition reveals a general constraint on space as the pure form of outer sense.

For Kant, as I understand him, what I see immediately when I recognize a plane figure as a triangle is guided by the very same rules I would use to construct an image of a triangle in imagination or to draw my own diagram of a triangle on paper. These are also the same rules I would follow to trace out a plane triangle with my finger or with the path of my whole body as I walked out a triangular pattern on a football field. The plane on which I conctruct or recognize the triangle must be oriented relative to my body in three-dimensional space. Even in the imagination, I think, Kant would claim, the plane on which a plane geometry construction is carried out is imagined as oriented in a three-dimensional space relative to a point of view.[29]

I think for Kant these general rules for recognizing or constructing any plane triangle support the auxiliary construction of lines *CD* and *CE* in the same plane. They also underwrite the intuitive reasoning whereby I am compelled to recognize that the sum of the interior angles equals the straight angle *BCE*. Since this construction and intuitive reasoning is supportable from the schema for the general concept of a plane triangle, the conclusion I reach for the figure under consideration must hold for any figure I could ostensively recognize to be a plane triangle.

3. *What about the non-Euclidean geometry of modern physics?*

I used Euclid's constructive proof that the sum of the interior angles of a plane triangle equals 180° to explicate Kant's account of geometrical constructions as an endorsement for the intuitive compulsion a proof like this provides. The fact that Kant cites this as a paradigm example of geometrical construction gives some support to those who claim that he was committed to the *a priori* application of Euclidean geometry to the space in which we can apprehend outer objects of experience. It is now well known that the sum of the interior angles of a plane triangle is a key mark discriminating between Euclid's geometry and the various non-Euclidean geometries of constant curvature. Geometries of positive curvature make the sum of the interior angles greater than 180°. This is clearly illustrated in Poincaré's model of plane Riemannian geometry in which the plane is identified with the outer surface of a Euclidean sphere. Geometries of negative curvature all make the sum less than 180°. These include the classic hyperbolic geometries of Lobachevsky and Bolyai. In a geometry with variable curvature the sum of the interior angles of a plane triangle marks the local curvature of the plane on which the triangle is constructed. If the sum is 180° then the space is locally Euclidean.

Kant has some good company if he was committed to Euclidean geometry – even among mathematicians who were aware of non-Euclidean geometries. I think it was an appreciation of just the sort of intuitive compulsion Kant's theory of constructions attempts to explicate that led Frege (1959) to claim that only Euclidean geometry fits our intuition and led Poincaré (1898) to suggest that Euclidean geometry ought to be retained even at considerable cost in additional complexity to physical theory. Nevertheless, I think that today most of us, children of the relativistic age as we are, would regard it as hopelessly Quixotic to continue to claim that Euclidean geometry is the correct geometry of the physical objects we meet in space-time. The weight of evidence is too solidly lined up behind modern physical theory. Does this not show, therefore, that the very foundation of Kant's empirical realism has been overturned by modern physics?

Strawson's (1966) attempt to save something of Kant's account of space – by making it apply to a merely visual geometry – will not do. This attempt and others like it (e.g. Walker's, 1978) which remove the clash with physical theory by giving up commitment to objective constraints on physical things, will not preserve the fundamental role of space as a framework within which appearances can generate an empirical realism.[30] I think Melnick (this

volume) is absolutely correct in his identification of the subject matter of Kant's account of geometry with the structure of our framework for meeting things outside us. Kant's space is the physical space we move our bodies around in. His straight lines correspond to rigid rods that can be rotated inside their boundaries and to paths of light rays.

I think one can keep this physical interpretation of the subject matter of geometry and keep enough of Kant's account of geometrical verification to give the constraints his empirical realism needs without flying in the face of modern physics. The key idea has been put forward by James Hopkins (1973) in an interesting paper attacking Strawson. It is this:

What we can establish by geometrical constructions is limited by our perceptual capacities.

Hopkins (p. 24, 25) points out that we could not take in any diagram that accurately represented the relative sizes and distances between two stars. Either the dots representing the stars would be too small to see or the distances would be so great we could not survey the diagram. I think what Hopkins is pointing out is correct and important. The limitations on what we can use diagrams to represent are quite significant. Even two parallel lines one centimeter apart – each say 0. 5 mm thick and 150 meters long could not be taken in by us. If we got far enough back to take in the end points we would be so far back that we would not be able to resolve the separate lines.

This shows that Euclid's parallel's postulate could not be established by any geometrical construction we could carry out. If Kant had claimed to be able to establish this postulate by constructions he would have violated his own basic injunction about extending concepts only valid for objects of experience beyond the limits of what we can experience. Any specification of what happens as parallel lines are extended indefinitely would correspond to an ideal of pure reason not to a principle constitutive of possible objects of experience. If we use the supervaluation method of representing commitment to the possibility of some such idealization then Kant's account of geometrical construction would commit him only to the envelope corresponding to a whole family of geometries each of which captured local constraints on three-dimensional shape and perspective up to tolerances provided by our perceptual capacities.

When you or I carry out our construction for the sum of the interior angles of a plane triangle the intuitive compulsion our result carries is not misleading, so long as we recognize that what we establish only holds up to tolerances provided by our perceptual capacities. Similarly, we really can constructively establish Ullman's various shape from motion theorems up to

such tolerances. Even though the constraints we get this way are vague and have vague limits on their vagueness, they have considerable bite. Lots of hypotheses are clearly beyond the tolerances allowed. For example, there could not be any rigid object, about as wide as my head, that would generate, at about arms length, the three orthographic projections specified in my salient illustration (Section IV, 1) of what Ullman's theorem rules out.

Such constraints capture a good deal of what Kant wanted from his account of geometry as a source of *a priori* knowledge. They depend on pervasive and accessible features of the actual capacities our sensory systems have and of the environment in which they have evolved to operate. These are very broad and deep features of what Wittgenstein called our form of life. They are not something we could change by adopting new social conventions. Nor could we find out tomorrow, on the basis of some new theory, that we have been wrong about these things all along. This does not imply that these constraints could never possibly change, only that to change them we would have to undergo rather gross changes in our bodies or their local environments. If the behaviour of light rays and measurably rigid rods were to change so dramatically that they become unreliable as indicators of shortest paths between macroscopic local locations, then perhaps the constraints would change. I believe Kant would say that such a change would be impossible.[31] Rather than just follow him in this. I want to point out the difference between having such physical changes actually begin to happen (which would be rather noticeable) and changing our theories about what has been happening all along. This difference points out an important sense in which geometrical constructions give us theory independent constraints on observation. It suggests to me that a Kantian alternative to some of the excesses of the last twenty years is to rush away from the idea of an observation theory distinction.

NOTES

* The earliest ancestor of this paper was a talk I gave at the Canadian Society for the History and Philosophy of Science in 1974. In 1978 I commented on Colin Turbayne at the Rochester Conference honouring Lewis Beck's retirement. This led me to develop the argument in Section I. The first written draft was in May 1982 and its first public presentation was at a conference at the University of Western Ontario in Spring of 1982.

A version was delivered as a lecture in my graduate seminar as Visiting Professor at Princeton in spring of 1983. Versions were also presented at a Duke Conference and at a Columbia University Philosophy Department colloquium. I am grateful for the insightful questions and comments received from many of the people who heard one or another of these presentations.

Special thanks are due to Robert Butts, Dan Garber, Ralf Meerbote, Calvin Normore, George Pappas, Margaret Wilson and Bas van Fraassen. Section II benefited from acute stylistic criticism generously provided by Paul Kirchner.

[1] Most (but not all) of the passages Turbayne uses to support Kant's commitment to this argument come from the fourth Paralogism (A 367–380) which was dropped from the second edition of the Critique. The new *Refutation of Idealism* (B 275–279) in the second edition uses a different argument (see Section 3). One can also, perhaps, quibble over some of the steps and the way they are arranged to bring out the affinity to Berkeley. Nevertheless, I think it is fair to say that Kant remained committed to something like this argument. Ralf Meerbote (correspondence with me) disputes this. I think he is correct unless special care is taken with step 2. See Note 8 below for a suggested interpretation of step 2 under which the argument is compatible with the refutation of idealism. It is under this interpretation (which differs from Turbayne's Berkleyean construal of (2) that I hold it plausible be assume that some such argument is important to Kant's Copernican revolution in philosophy.

[2] Several writers, e. g. N. K. Smith (Commentary, p. 156) have taken Kant's accusations of illusionism as evidence that he misunderstood Berkeley's position. What George Miller (1973, pp. 316–322) has called the traditional view of the relation between Kant and Berkeley would explain these apparent misunderstandings on the hypothesis that Kant only knew Berkeley's work through distorted second hand sources. Turbayne (pp. 225–227), Miller (*op. cit.*) and Henry Allison (1973, pp. 43–45) have made it plausible to assume that Kant had far more access to Berkeley's work than the traditional view would allow. In particular they point out that a German translation of Berkeley's dialogues was readily accessible to Kant.

The hypothesis that Kant actually read the dialogues allows one to entertain the view that Kant's reference to the 'good Berkeley' in his B 70 passage

> we cannot blame the good Berkeley for degrading bodies to mere illusion:

which Turbayne finds evidence of animus and Allison of condescension is really only irony obtained by applying to Berkeley the very same rhetorical device he applies to Hylas (the defender of common sense realism) in the dialogues –

> *Phil:* "Have patience, good Hylas, and tell me once more whether there is anything immediately perceived by the senses expect sensible qualities. I know you asserted there was not; but I would now be informed whether you still persist in the same opinion."

[3] Turbayne, Margaret Wilson (1971), Goerge Miller (1973) and Henry Allison (1973) all point out that in Kant's day Berkeley's position was regarded very unsympathetically.

[4] George Pappas brought to my attention James Comman's (1973) defence of the idea there can be laws connecting sense data. I believe this defence will not work if sense data are to be construed as incorrigible subjective contents of experience (see Note 14).

[5] It is not surpising that Sellars, the author of the objections to phenomenalism I have been considering, should take such rules governing what he called point-of-viewish aspect of perception as the key to an interpretation of Kant's transcendental idealism. In 'Kant's Transcendental Idealism' 1975 and in 'The Role of Imagination in Kant's Theory of Experience' 1978, Sellars proposes what I take to be just the sort of account I shall defend here. Indeed, this paper can be well construed as an attempt to make some of the details of this kind of account more explicit and to document more extensively its textual support in Kant's writing.

[6] Margaret Wilson (1971), George Miller (1973) and Henry Allison (1973) all argue impressively against Turbayne that it was reasonable for Kant to draw the conclusion that Berkeley's

treatment of space renders his position unable to support an empirical realism. I agree with most of what these writers have to say and offer these additional arguments in support of the view that Kant's conclusion about Berkeley's position is true as well as having been reasonable for him to draw.

[7] John Pollock (1974) and Hilary Putnam (1981) are but two of many recent examples.

[8] It is well known that Kant explicitly addressed this paralogism against Descartes' position (A 368), so it is not too surprising that the assumptions of the demon hypothesis should correspond to the assumptions leading to the skeptical nadir of the transition from transcendental realism to transcendental idealism.

Some care must be taken with step 2. For one thing, it will turn out that for Kant, unlike Berkeley, immediate awareness need not mean unchallengeable. For another, Kant will distinguish between subjective and objective contents of representations. A Berkelean construal of step 2 on which the content in question is subjective and immediacy implies not open to challenge is what corresponds to Wilson's contention 2. On this construal, I shall argue, Kant's transcendental idealism gives up contention (2) as well as contention (1). If step two is interpreted so as to include objective content and immediate awareness so as to allow for corrigibility then Kant does not give up step 2. Under this interpretation (in which step (2) is not the same as Wilson's contention (2)) Kant's Fourth Paralogism argument, in which step (2) is retained, is quite compatible with his refutation of idealism, which I shall argue rejects Wilson's contention (2).

[9] Berkeley's subjective idealism is a salient example in a long tradition of phenomenalistic empiricism that is characterized by the attempt to ground all acceptable knowledge claims is sensations or some kind of incorrigible data base in experience. This tradition, which includes Hume, Mill, Russell at some stages, the Carnap of the *Aufbau*, and C. I. Lewis, is still alive today in the work of R. M. Chisholm and John Pollock. All of these writers including the most sophisticated agree with Berkeley in holding to contention (2) and most of them give up contention(1) in some way or other. In recent years, perhaps to a great extent due to the influence of Wittgenstein, this idea of a secure data base incorrigible claims about subjective contents of experience has lost power. As this has happened more and more Kant scholars have opted for objective rather than subjective readings of transcendental idealism. If my interpretation is correct then this has been a good trend, for Kant' position always was distinctively different from Berkeley's in that his appearances should never have been construed as incorrigible data.

[10] Turbayne (pp. 232–3) considers this skeptical idealism (step 4) to be the first stage of the solution to the skeptical problem. I think Kant considered this position as no better than what Turbayne calls the deepest skepticism of step 3.

[11] Note that, on Turbayne's version of it, Berkeley's position would also have to allow that judgments about external objects could be mistaken. Even if my judgment about the subjective content I now have were incorrigible my judgment that uniformities appropriate to the claim that there is a real dagger there obtain is subject to error.

[12] Of the two interpretations of Kant's refutation of idealism argument suggested by Wilson (1972, pp. 604–605) this is the one that she grants would make trouble for her. It is also the one that best coheres with what Kant says the argument proves (B 275).

[13] Barry Stroud (1968) also makes interesting comparisons between Kant's refutation of idealism argument and Wittgenstein's private language argument. Unlike Sellars and Bennett, however, he did not actually propose an argument for Kant's conclusion.

[14] This collapse into the solipsism of the present moment provides an additional compelling argument against the hypothesis that there are laws about sense data construed as incorrigible

subjective reports that could be learned empirically from observed regularities in one's subjective experience (See Note 4.)

I explore connections between the foregoing interpretation of the Refutation of Idealism and the Second Analogy Passages on the distinction between subjective and objective succession in Harper (1984). That paper also uses the nice passage from Wilson as a paradigm of the Demon argument and the Cartesian assumptions it requires. I first saw this kind of interpretation of Kant's Refutation of Idealism, where the key argument is the failure of subjective idealism to support objective truth conditions about past subjective experiences, in Jonathan Bennett (1966). Paul Guyer (1983) has recently provided an extensive discussion of the origin and interpretation of Kant's Refutation of Idealism which also makes the core of the argument depend on these considerations.

[15] It is worth remarking that Kant uses 'representation', and each sub-heading on this list, ambiguously as a token term to refer to particular mental episodes (represent*ings*) and also as a type term to specify a kind of representation *qua* – what it represents and how it represents it. The important use is the type use. I shall attempt to restrict my use of 'representation' to it and reserve represent*ing* for the token use. When, for example, I speak of my empirical intuition of this coin I shall be speaking of a kind of representing which can be specified as a representing of this coin under certain perceptual circumstances. According to this usage the same intuition could have tokened by another person or have failed to be tokened at all should it have been that someone else or no one at all had satisfied the relevant perceptual circumstances.

[16] Van Fraassen (Lecture at University of Western Ontario, fall term 1981) had recently suggested that one's inability to believe there is no ball in his hand when he is confronted with it does not show that belief is not voluntary any more than the fact that one cannot steal when he is in his own bathtub surrounded by only his own possessions shows that stealing is not voluntary. According to van Fraassen circumstances can sometimes constrain belief. Presumably, one of the most important kinds of constraint is provided by perceptual circumstances. It is to our relative inability to override these circumstances that I point when I speak of the independence of perception.

[17] Sellars (e.g. 1963a) has emphasized the idea that entities of various kinds could play in thought a role suitably analogous to the role placed by a sentence, e. g.

This is a quarter-shaped object before me,

in English. My proposal here is designed to be in the spirit of his general use of dot quotes and his own accounts of Kant's intuitions (e. g. 1968, Chapt. I, 1975, 1978).

[18] This account of appearances is very close to Sellar's account of what we see *of* an object (e.g. 1981, Sections 15–24). It is also very close to Gibson's account of 'affordances' for human perception (Gibson, 1966, 1979). I hope to explore some of the connections with Gibson's work in a later paper. Indeed, I expect that some of Kant's use of geometry to ground his account of appearances as undetermined objects of empirical intuition can help answer some of the objections (e.g. in J. A. Fodor and Z. W. Pylyshyn, M. I. T. Occasional paper # 12) that have been raised against Gibson.

[19] One of the salient differences between colors and such geometrical properties as shape is that the same geometrical property can often be presented to touch as well as to sight. This legitimate and important integration of sense realms, noted as early as Aristotle, provided motivation for the infamous distinction between primary and secondary qualities.

According to Berkeley (Theory of Vision, 121–146) vision is only presented with colour, and colour is never presented to touch, so that these two sense realms are entirely distinct. He insists

that the shapes presented to touch and the shapes presented to sight are entirely different species related much as the combinations of writer's letters are related to the sounds of speech. In 'The Perception of Shape' (1983a) David Sanford has offered an excellent exploration of and defence for the claim that the shapes we see are the same as the shapes we touch. This is the best discussion of the issue I know of.

[20] In 'Kant's Empirical Realism and the Distinction Between Subjective and Objective Succession' (Harper, 1984) I presented a line by line interpretation of this paragraph. This section of the present paper is, mostly, a summary of the main points in that interpretation; however, it does contain some additional remarks I hadn't thought to make before.

[21] Beck's paper contains an admirable brief gloss of Kant's entire third paragraph in *B*. A good deal of my longer interpretation was cast in the form of commentary and expansion on Beck's gloss.

[22] Beck distinguishes three distinct versions of Kant's empirical sense of 'appearance'. See (Harper, 1984) for my exposition of their relation to my explication of appearance as undetermined object of an empirical intuition.

[23] This gloss of the two occurences of 'it' in the translation was suggested to me in correspondence by Lewis Beck. It is a salient part of the reading he provides in Beck (1978), pp. 144–146).

[24] See van Fraassen (1966) for the basic account of supervaluation. See Thomason (1973) and Hans Kamp (1981) for applications of supervaluations to problems of indeterminacies generated by vagueness.

[25] *The Metaphysical Foundations of Natural Science* suggests that Kant himself would opt for a plenum account based on postulated centers of inverse square attractive forces and inverse cube repulsive forces.

[26] I have only argued that Kant's own defence of empirical realism is based on his appeal to *a priori* spatial constraints. I have not attempted the ambitious task of showing that an empirical realism that does not presuppose such constraints is impossible. Nevertheless, I hope the arguments of this section will make it implausible to suppose that one can recover an empirical realism without presupposing some such constraints. More importantly, I hope to have shown that the sort of constraints Kant's defence requires are not so very implausible to presuppose.

[27] Orthographic projection is not an accurate representation of the visual information afforded to an observer surveying a relatively large object before her. Ullman explores several other projection schemes. One is a perspective projection scheme according to which three views of five elements are mostly sufficient to uniquely specify the configuration and relative motion. This scheme is better at discriminating reflections in the image plane, but not so efficient at ruling out other alternative configurations and motions. The most sophisticated model of a projection scheme for human vision he considers is a polar-parallel projection scheme. Points corresponding to local texture patterns on a large surface are treated as approximately parallel projections to give detailed information about the local surface shape, while the larger structure is pinned down by polar-projection of these various local textured areas. This scheme apparently provides a fair approximation to the strengths and weaknesses of actual human visual discrimination.

[28] Beck (1956) and Parsons (1969) have made a convincing case that Kant interprets such constructions as *real definition* of a geometrical concept. Real definitions are not analytic nor are they to be merely conventional stipulations. I have tried to make it plausible that geometrical constructions can play such an exalted role.

[29] David Sanford (1983b) has an interesting discussion of the commitment to orientation relative to a point of view of a visual field. I think Kant would agree with this and would extend the point to imagination as well as perception.

[30] Reid's Geometry of Visibles has been resurrected as one of a number of proposals for construing visual geometry as non-Euclidean (Angell, 1974; Daniels 1972). Strawson's attempt to save Euclidean geometry by dividing visual geometry from physical interpretations plays right into the hands of these advocates for non-Euclidean geometries of visual experience.

[31] I do think that it is rational now to proceed as though such changes could not happen. Indeed I think that we all really do proceed this way. We cannot help but make them into conceptual commitments for us. I also think that conceptual commitments can be rationally changed (Harper, 1978).

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