KANT ON THE A PRIORI AND MATERIAL NECESSITY

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I. "A Priori", "Empirical", and "Necessary"

1. "A Priori" vs. "That which is Borrowed from Experience"

In the Introduction to the first edition of KRV[1] Kant characterizes a priori knowledge in the following passage:

> Experience tells us, indeed, what is, but not that it must necessarily be so, and not otherwise. It therefore gives no true universality; and reason, which is so insistent upon this kind of knowledge, is therefore more stimulated by it than satisfied. Such universal cognitions (**Erkenntnisse**), which at the same time possess the character of inner necessity, must in themselves, independently of experience, be clear and certain. They are therefore entitled knowledge (**Erkenntnisse**) **a priori**; whereas, on the other hand, that which is borrowed solely from experience is, as we say, known only **a posteriori** or empirically (**KRV** A1-2).

Kant tells us that such universal cognitions as also possess the character of inner necessity are to be entitled a priori, since they must in themselves, independently of experience, be clear and certain. This suggests a core idea of a priori knowledge as that which *independently of experience is clear and certain*. The main contrast is with an idea of *merely empirical knowledge* characterized as knowledge *borrowed solely from experience*. Kant tells us that this merely empirical knowledge can include no knowledge of necessity and, therefore, no true universality.

This initial contrast does not seem to be an exhaustive division of knowledge. Consider the first part of Proposition II from Book III of Newton's *Principia*:

1/ All translations of **KRV** are from Smith (1929), sometimes modified.

R. E. Butts (ed.), Kant's Philosophy of Physical Science, 239–272.

> That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the sun... (Cajori 1962, p. 406).

Our knowledge of this proposition is surely not properly described as "borrowed solely from experience", nor would we describe it as "clear and certain independently of experience...". I shall argue that this proposition, together with other propositions Newton claims to deduce from phenomena, and indeed, perhaps, Newton's principle of universal gravitation itself, count for Kant as interesting examples of mixed items of knowledge inferred from a combination of experience and strictly universal principles. I shall also argue that according to Kant's official characterization of necessity in the Postulates of Empirical Thought such mixed items of knowledge count as necessary.

2. Universality and the Merely Empirical

Many commentators take Kant to intend an exhaustive classification, so that the core sense of "a priori knowledge" as "clear and certain independently of experience" may be equivalently characterized as "non-empirical knowledge". D. P. Dryer (1966, p. 30) puts the point as follows:

> By "a priori" judgments he [Kant] simply means non-empirical judgments. His word 'a priori' is but a synonym for 'non-empirical'.

On Dryer's explication judgment rather than knowledge is basic. An item of empirical knowledge would be the content of an empirical judgment known to be true. For our present purposes, the important thing about this classification is that Dryer makes it exhaustive by interpreting 'empirical' broadly.

> Some judgments can be verified merely by observing what they are about; other judgments can be verified by inference from what is observed. Whether it can be verified directly or indirectly, Kant means by an empirical judgment a judgment which could be made out to be true or false **only** by some recourse to observation (Dryer 1966, p. 31).

On this classification those mixed items of knowledge inferred from some combination of experience and strictly universal principles will count as empirical.

This broad construal of 'empirical' is apparently supported by Kant's often repeated insistence that a priori cognitions enter into experience itself. Indeed, the paragraph immediately following the first edition text we have been considering makes this very point.

> Now we find, what is especially noteworthy, that even into our experiences there enter cognitions [**Erkenntnisse**] which must have their origin **a priori**, and which perhaps serve only to give coherence to our sense representations. For if we eliminate from our experiences everything which belongs to the senses, there still remain certain original concepts and certain judgments derived from them, which must have arisen completely **a priori**, independently of experience, in as much as they enable us to say, or at least lead us to believe that we can say, in regard to the objects which appear to the senses, more than mere experience would teach--giving to assertions true universality and strict necessity (**KRV** A2).

The first part of this passage suggests that, according to Kant, not even such paradigmatically empirical judgments as my observation that there is an object shaped approximately like a rectangular solid over there (made as I look at a book on my desk) would count as "borrowed solely from experience". It also suggests that what might count as "borrowed solely from experience" is what belongs to the senses alone.

One of the avowed aims of **KRV** is to attack an empiricist conception of experience that would limit empirical knowledge to what we receive through the senses. So, there is much to be said for an interpretation that would fix the extension of Kant's use of 'empirical' broadly to include mixed items as empirical knowledge. The narrow construal evoked by the slogan "borrowed solely from experience" seems more in keeping with Kant's empiricist opponents than with his own considered views; nevertheless, it seems to be some such narrow construal he intends when, as in the last part of the passage, he contrasts the a priori, as a source of true universality and strict necessity, with what *mere experience* can teach us.

In the passage in the second edition of **KRV** where he expands on his characterization of strict universality as a mark of the a priori Kant further specifies his contrasting sense of 'empirical' by limiting it to the *merely assumed* and *comparative universality* that can be given through *induction*.

> Experience never confers on its judgments true or strict, but only assumed and comparative **universality**, through induction. We can properly only say,

therefore, that as far as we have hitherto observed, there is no exception to this or that rule (B3-4).

He also suggests that

Empirical universality is only an arbitrary extension of a validity holding in most cases to one which holds in all, for instance, in the proposition, 'all bodies are heavy' (B4).

So, we have here in the proposition that all bodies are heavy what Kant takes to be an example of the sort of empirical generalization we infer through induction. We also have his characterization of inductive inference as *an arbitrary extension of a validity that holds in most cases to one that holds in all.*

This characterization of inductive inference reminds one of Hume's account of the natural inferences we make when we generalize from observed constant conjunctions (Hume 1977, §V). We cannot help but make such generalizations; but, as far as merely empirical knowledge goes, we can properly only say that as far as we have hitherto observed there is no exception to this rule. As Hume points out, we need to rely on such natural inferences in our practical life (Hume 1977, §XII). To allude to an example of Kant's (B2), we do not undermine the foundations of our houses, even if our strictly empirical evidence only covers those unsupported bodies we have actually observed to fall.

Kant's characterization of inductive generalization as an arbitrary extension yielding only an assumed or comparative universality also suggests a Humean critique of the sort of idealization to be found in Newton's treatment of Kepler's laws of planetary motion (and indeed in scientific curve fitting inferences generally). Speaking merely empirically we should say only that such and such a finite set of astronomical observations have been reported, but, Newton (Phenomenon V, *Principia*, Bk. III) takes as a phenomenon that the five known primary planets describe by radii drawn to the sun areas proportional to the times of description. This is a generalization that covers infinitely many occasions lying between those covered by the finite set of observations. It also extends far into the past and indefinitely far into the future. Kant's remarks suggest that in so far as this proposition is to be regarded as a merely empirical law (supported only by induction from these observations) it should count as an arbitrary extension of a validity holding in these few observed instances to a generalization that purports to cover this infinity of intended instances.

If, as Kant's discussion suggests, the "merely empirical" is to be limited to what inferences from experience Hume thought we could rationally justify, then mixed items such as Newton's inference to centripetal orbital forces will not count as "merely empirical". Indeed, on this very strict construal most of the inferences we make in everyday life as well as science will also fail to pass muster as "merely empirical". A more liberal interpretation, which would remain true to Kant's characterization of the "merely empirical" as what induction can teach, is forthcoming if we bypass Hume's skeptical doubts and let the results of what he called our "natural inductive inferences" count as empirical. I think an analogy to the contrast between even such a liberal view of induction and natural kind inferences in contemporary science can illuminate Kant's contrast between the merely empirical results of induction and the sort of universality that, he claims, requires appeal to the a priori.

Let us suppose that the merely empirical allows appeal to whatever is required to support probabilistic induction for a reasonably generous class of projectible predicates including specific gravity values. Suppose we measure the specific gravity of one sample of a given alloy of copper (for example, commercial bronze 90 Cu, 10 Zn), then another, and another, etc. In each case let us say the measured value is 8.8 to within some given tolerance for measurements errors. In contemporary Bayesian terms we can increase our rational degree of belief that an as yet unexamined sample has specific gravity 8.8 as we examine more and more positive instances, provided our prior probability assignments satisfy some relatively weak constraints. Let us allow quite daring constraints: for example, an equal prior probability assignment between the generalization and an alternative hypothesis that would have specific gravity values independently and randomly distributed. This would allow our rational degree of belief in the generalization, as well as the corresponding singular claim, to increase as the evidence of positive instances accumulates; but, we would not be able to push either of these probabilities to certainties, no matter how many positive instances we observed. Thus, there is a plausible sense in which we can say that even such liberal inductive inferences can provide only an assumed and comparative universality.

What do we actually do? We make a natural kind inference. We assume outright that all samples of a given alloy of metal have the same specific gravity. To whatever tolerance we deem ourselves to have measured the specific gravity of the given sample to be 8.8, we deem ourselves to have established 8.8 as the specific gravity of all samples of that alloy. Perhaps this is an example of a sort of universality that Kant would tell us cannot be learned through mere experience. Apparently, it goes beyond what even quite liberal accounts of probabilistic induction from instances can deliver. It is also typical of contemporary scientific practice. We would not have much left of what we call science if we attempted to do without such natural kind inferences.

This natural kind inference goes through only relative to theoretical commitments that deem specific gravity to be essential to the kind of alloy a sample of metal is made of. We would not be able similarly to infer the shape of other samples from the shape of a given sample. Once a theoretical conception is entrenched in the role of specifying the nature of such kinds as our copper alloy it may be reasonable to treat it as relatively immune to revision. It may well take on a status akin to what Putnam has called the "contextual a priori" (Putnam 1983, p. 95), a status ensuring that it could be given up only after formulation of a rival conception that does an equally good or better job of making the relevant range of phenomena intelligible and providing a framework for guiding our practical dealings with those phenomena.

If we take seriously the role of such contextually a priori commitments we would seem either to have to count them as non-empirical or to extend our concept of empirical knowledge beyond the bounds of even very liberal accounts of probabilistic induction from instances. Therefore, I think attention to natural kind inferences suggests that historical and contemporary scientific practice may be more like Kant's view, on which a priori commitments support inferences to universal claims, than it is like even quite sophisticated inductive empiricisms.

3. Necessity

In Section II of the Introduction to the second edition of KRV Kant expands on his suggestion that necessity and universality cannot be learned from experience alone.

> Experience teaches us that a thing is so and so, but not that if cannot be otherwise. First, then, if we have a proposition which in being thought is thought as **necessary**, it is an **a priori** judgment; and if besides, it is not derived from any proposition except one which has the validity of a necessary judgment, it is an absolutely **a priori** judgment (B3).

Apparently, the salient point here is that mere experience only teaches us contingencies, so that to think a proposition as necessary is to make an a priori judgment. Another point of some interest to us is the distinction between merely a priori judgments and *absolutely* a priori judgments. This distinction presupposes that one could correctly think a proposition as necessary, thus making an a priori judgment, even though this judgment is derived (in part) from propositions that do not have the validity of necessary judgments.

When we look more closely at Kant's characterization of strict universality as a mark of the a priori we find some tension.

> Thus, if a judgment is thought with strict universality, that is, in such a manner that no exception is allowed as possible, it is not derived from experience, but is valid **absolutely a priori** (B4; emphasis supplied).

Here Kant tells us that if a judgment is thought with strict universality, that is, so that no exception is allowed as possible, then it is valid *absolutely* a priori; but, he has just told us that a judgment is valid *absolutely* a priori only if besides having itself the validity of a necessary judgment, it is not derived from any proposition that is not also necessary. These two points together allow for a judgment that is necessary but has less than strict universality. The tension arises from the apparent difficulty of understanding how a proposition can be thought as necessary without also thinking it in such a way that no exception is allowed as possible. Necessity and possibility are standardly construed as inter-definable. Given a notion of possibility, the corresponding notion of necessity specifies that a proposition is necessary just in case its negation is not possible.

This tension is resolved when we examine Kant's official characterization of possibility and necessity in the Postulates of Empirical Thought.

> 1. That which agrees with the formal conditions of experience, that is, with the conditions of intuition and of concepts, is **possible**.

2. That which is bound up with the material conditions of experience, that is, with sensation, is **actual.**

3. That which in its connection with the actual is determined in accordance with universal conditions of experience, is (that is, exists as) **necessary** (KRV A218/B265-266).

We have here two notions of necessity. First, there is the sense in which a proposition is necessary if its negation is not possible in the sense explicated in Postulate One. Something necessary in this unconditional sense would follow from the universal conditions on objects of experience alone, without appeal to any specific empirical content. Second, there is Kant's official category of necessity, which is a conditional necessity derived from universal conditions of experience together with some specific actuality. This kind of necessity may require appeal to some specific empirical content as well as to the strictly universal formal conditions on possible objects of experience.

The tension is resolved by interpreting a judgment that has the second but not the first kind of necessity as a priori but not as absolutely a priori. Such a judgment would not be absolutely a priori, because it would be derived, in part, from empirical knowledge that some specific actuality obtained. It would not be strictly universal, because exceptions would be possible in that its negation would not contradict the universal conditions of experience. Nevertheless, one who was in a position to know the relevant actuality would also be in a position to rule out any such exception. According to Kant, he would be in a position to ascertain that the knowledge claim in question had the validity of a necessary judgment. Therefore, if necessity is to be a mark of the a priori this judgment would count as a priori, even though it requires some empirical evidence to derive it.

Kant makes the following explanatory remarks concerning the Third Postulate:

Lastly, as regards the third postulate, it concerns material necessity in existence, and not merely formal and logical necessity in the connection of concepts. Since the existence of any object of the senses cannot be known completely **a priori** but only comparatively a **priori**, relatively to some other previously given existence; and since, even so, we can then arrive only at such an existence as must somewhere be contained in the context [**Zusammenhange**, **connection**] of the experience, of which the given perception is a part, the necessity of existence can never be known from concepts, but always only from connection with that which is perceived, in accordance with universal laws of experience. Now there is no existence that can be known as necessary under the conditions of other given appearances, save the existence of effects from given causes, in accordance with laws of causality. ...Necessity concerns only the relations of appearances in conformity with the dynamical law of causality and the possibility of inferring a priori from a given existence (a cause) to another existent (the effect) (A227-228/B279-280).

Kant is making it clear here that the necessity explicated in the Third Postulate is that with which an effect is necessitated by its cause. In the Second Analogy Kant argues for the principle that

Everything that happens, that is, begins to be, presupposes something upon which it follows according to a rule (A190).

In the course of the argument, he tells us,

In conformity with such a rule there must lie in that which precedes an event the condition of a rule according to which this event inevitably and necessarily follows (A193/B239).

and later,

...if the state which precedes is posited, this determinate event follows inevitably and necessarily (A198/B244).

These are but two of the many passages where Kant commits himself to the claim that if *A* causes *B* then the specific connection between *A* and *B* is necessary.

As we have seen, the sense of 'possibility' explicated in Postulate One generates a corresponding sense of 'necessity' according to which a proposition is necessary just in case its negation is ruled out by the formal conditions on possible objects of experience. This unconditional sense of 'necessity' also determines the conditional sense of 'necessity' explicated in the Third Postulate, provided that the *universal conditions of experience* referred to in the Third Postulate are the same as the *formal conditions of experience* referred to in the First Postulate. Under this identification we have that

actuality *A* necessitates *B*

in the sense of the Third Postulate only if the hypothetical judgment

if *A* then *B*

is unconditionally necessary in the sense of the First Postulate. The combination *A* and not-*B* would have to be ruled out as impossible by the formal conditions on possible objects of experience if the actuality of *A* is to necessitate *B* in the sense of the Third Postulate. This would seem to require that the specific causal law corresponding to the rule connecting a given cause and effect would have to be strictly universal and thus absolutely a priori.

This conclusion can be avoided if the universal conditions referred to in the Third Postulate are not the formal conditions referred to in the First Postulate. One way to do this is to postpone the requirement of unconditional necessity by allowing the hypothetical 'if *A* then *B*' to have, itself, only conditional necessity; that is, to have its necessity be relative to some further actuality *A'* so that only the hypothetical hypothetical

if *A'* then, if *A* then *B*

would count as unconditionally necessary. Perhaps this sort of postponement could not be extended indefinitely. If not, then it would seem that more general hypotheticals sufficient to ground the detailed connections of specific causal laws in actualities would have to be absolutely a priori.

4. The A Priori as Independent of Experience

In the second edition of KRV Kant expanded on his core idea of the a priori as "clear and certain independently of experience" with his famous characterization of a priori knowledge as "absolutely independent of all experience".

> In what follows, therefore, we shall understand by a **priori** knowledge not knowledge independent of this or that experience, but knowledge absolutely independent of all experience. Opposed to it is empirical knowledge, which is knowledge possible only a **posteriori**, that is, through experience (B2-3).

This extremely strong characterization of the a priori would apply, it would seem, only to the sense of "absolutely a priori" knowledge we have been discussing. It would not, apparently, be applicable to propositions derived in part from empirical knowledge that some actuality obtains, as are the necessary propositions picked out by Kant's official characterization of 'necessity'. We also see in this passage a more liberal characterization of 'empirical knowledge' than that suggested by Kant's discussion of necessity and universality as marks of the a priori (B3-4). On this liberal characterization the necessary propositions characterized in the Third Postulate count as empirical as do all mixed items of knowledge.

In the preceding paragraph Kant indicated that his main concern in offering such a strong characterization is to make it clear that he is not intending such common usages whereby, for example, a man can be said to know a priori that his house will fall if he undermines its foundation.

> The expression '**a priori**' does not, however, indicate with sufficient precision the full meaning of our question. For it has been customary to say, even of much knowledge that is derived from empirical sources, that we have it or are capable of having it **a priori**, meaning thereby that we do not derive it immediately from experience, but from a universal rule--a rule which is itself, however, borrowed by us from experience. Thus we would say of a man who undermined the foundations of his house, that he might have known **a priori** that it would fall, that is, that he need not have waited for the experience of its actual falling. But still he could not know this completely **a priori**. For he had first to learn through experience that bodies are heavy, and therefore fall when their supports are withdrawn (B3).

Even though I can know that if I undermine its foundation my house will fall independently of the experience of actually undermining it, this knowledge depends on my knowing some generalization like 'bodies are heavy and, therefore, fall when their supports are withdrawn', which, he tells us, had to be borrowed from experience.

As we have seen, Kant used the example 'all bodies are heavy' to illustrate empirical universality as an arbitrary extension of a validity holding in most cases. This suggests that what prevents the man's knowledge that his house will fall from counting as truly a priori may be only that the universal rule he applies has only the assumed and comparative universality of empirical induction. This man's knowledge that his house will fall does not have the validity of a necessary judgment even in the weaker sense explicated in the

Third Postulate. If Kant wants his strong characterization to rule out such cases only, then it may well be that we should look for an interpretation of "absolutely independent of all experience" that would allow mixed items derived from actualities together with strictly universal rules to count as a priori.

Kant had already suggested that experience may be required to awaken our faculty of knowledge into action.

> For how should our faculty of knowledge be awakened into action did not objects affecting our senses partly of themselves produce representations, partly arouse the activity of our understanding to compare these representations, and, by combing or separating them, work up the raw material of the sensible impressions into that knowledge of objects which is entitled experience? (B1).

So, his characterization of a priori knowledge as "absolutely independent of experience" should not be taken to rule out knowledge that requires some experience to appropriately awaken the faculty. Among propositions Kant would claim we can know a priori in his intended strong sense are:

(1) 4,367 + 25 = 4,392

(2) Every alteration has a cause;

therefore, the fact that some experience with arithmetic is required to know (1) and that (2) might not be known by someone innocent of the right kind of philosophical experience does not make either fail to be absolutely independent of experience in the sense Kant intends.

These examples illustrate a distinction Kant makes among propositions knowable a priori. According to Kant the first is a *pure* proposition because it involves no admixture of anything empirical, while the second is not pure because alteration is a concept that can be derived only from experience.[2]

2/ The distinction between pure and impure a priori knowledge is somewhat delicate. On the very next page (B4-5) Kant refers to the judgment that 'every alteration must have a cause' as a pure a priori judgment, and characterizes such judgments as those that are necessary and in the strictest sense universal. This would make the distinction the same as that between absolutely a priori and merely a priori judgments. Presumably, however, Kant means to distinguish among absolutely a priori judgments

A priori knowledge claims [**Erkenntnisse**] are entitled pure when there is no admixture of anything empirical. Thus, for instance, the proposition 'every alteration has its cause', while an a priori proposition, is not a pure proposition, because alteration is a concept which can be derived only from experience (B3).

This reinforces the lesson that experience may be required to properly awaken the faculty for knowledge of a priori propositions. It tells us that some concepts can be derived only from experience, and that a proposition formed from such concepts could, nevertheless, be known a priori even though a great deal of experience, perhaps a long history of scientific development, would be required to get us to form the concepts in it. So, perhaps Kant's claim of a priori status for his versions of the laws of motion in MAN is less extravagant than it has often been taken to be.

II. Newton's Inference

1. The Universal Rule

I suggested above that Newton's inference to the central direction of forces holding the primary planets in their orbits (the first part of Proposition II, Book III of *Principia*) is an interesting example of a mixed item of knowledge. Later I suggested that such mixed items count as necessary in the sense explicated in Kant's Third Postulate of Empirical Thought, provided that they follow from some actuality in accordance with some appropriately strict universal conditions of experience. Let us now examine the example in more detail. The relevant actuality will have to be what Newton takes to be the phenomenon that these planets satisfy Kepler's law of areas (areas swept out by radii drawn to the sun are proportional to the times of description). The appropriately strict universal conditions of experience will have to be those ap-

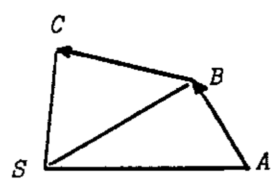
those that are pure, on the basis of a distinction between pure and empirical concepts. The pure concepts are, apparently, those that can be constructed in what Kant would regard as pure mathematics, together with those corresponding to the purely logical forms of judgment. Concepts from geometry and arithmetic would certainly count as pure. Concepts generated from application of mathematics to actual change, like Kant's explication in MAN of motion as change of external relation to a given space (p. 482), apparently would count as empirical. See Friedman (1985, p. 482n) for an interesting suggestion.

pealed to in Newton's proof of the theorem he uses to infer the central direction of the orbital force from this phenomenon.

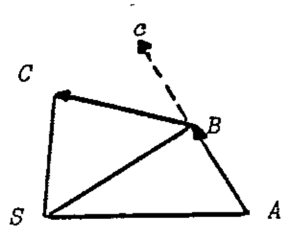
Let us begin with the theorem (Proposition II/Theorem II).

Every body that moves in any curved line described in a plane, and by a radius drawn to a point either immovable, or moving forwards with an uniform rectilinear motion, describes about that point areas proportional to the times, is urged by a centripetal force directed to that point (Cajori 1962, p. 92).

Consider the following diagram:



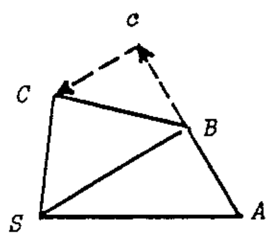
Assume a body uniformly traverses line *AB* in a given unit of time. At *B* let it undergo a force that instantaneously deflects it to a new velocity such that it uniformly traverses line *BC* in a second equal unit of time. Suppose that equal areas are swept out in these equal times by radii drawn to center *S*; that is, suppose triangle *SAB* equals triangle *SBC*. Our problem is to find the direction of the force that acted at *B*. Construct extension *Bc* equal to line *AB*.



By Newton's first law of motion, line *Bc* is the trajectory the body would have traversed in the second time interval had no force deflected it.

Newton's first law is Kant's Proposition 3 in his chapter on Mechanics in **MAN** (p. 543). According to Friedman, this law follows by what is, in effect, a transcendental argument from the absence of any independent empirical criterion for measuring times (Friedman 1986, pp. 34-35). If what can count for us as equal times are the times in which bodies undergoing uniform inertial motion traverse equal distances, then nothing could count for us as the empirical truth of an instance in which this first law can be violated.

By Newton's parallelogram of forces corollary (Cor. I, Bk. I) to the laws of motion, *BC* is composed of the motion of *Bc* together with a motion at *B* in the same direction and magnitude as the line connecting *c* to *C*. Therefore, the line from *c* to *C* gives the direction in which the deflecting force acted at *B*.



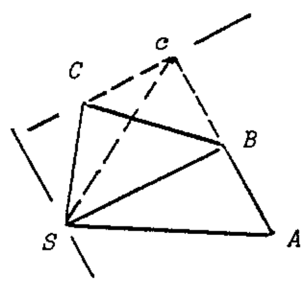
This parallelogram construction for compositions of motions builds in commitment to invariance of distance with respect to Galilean transformations from one relative space to another. This is the part of classical kinematics that Einstein revised when he developed the Special Theory of Relativity. Today, we do not regard it as a good candidate for a priori status, since we regard it as having been empirically refuted by contemporary physics.

Kant's version of the parallelogram construction builds in the same commitment to Galilean invariance as does Newton's. Indeed, this commitment is perhaps even more immediate for Kant because he explicitly requires that one of the motions to be composed be represented by an appropriate motion of a relative space in relation to the space in which the other motion is represented (**MAN**, p. 490). Kant also tells us that

To construct the concept of a composite motion means to present a priori in intuition a motion insofar as it arises from two or more given motions united in one movable thing (*MAN*, p. 486).

Therefore, there is good reason to suppose that he would regard this step in Newton's proof as having as good a claim to a priori status as has a result of a purely mathematical construction. One question we must consider is whether we can deem this kinematical construction to have provided some appropriately strict universality.

The next step in the proof is to show that line *cC* is parallel to *BS*. This is a problem in geometry. Construct line *Sc*.



Triangle *SAB* equals triangle *SBc*, since they have equal bases *Bc* and *AB* and have equal heights to a parallel to these bases *ABc* erected at *S*. We assumed triangle *SAB* equal to *SBC*; therefore by transitivity of equality we have triangle *SBc* equal to triangle *SBC*. (Kant would surely count the transitivity of equality as a priori. Indeed, he would very surely include it among "those few propositions presupposed by the geometrician which are really analytic" (KRV B16).) These two equal triangles *SBc* and *SBC* share the same base *BS*; therefore, they have the same height to a parallel to this base, so the line joining vertex *c* to vertex *C* is this parallel to *BS*.

Here we have a commitment to the application of a construction in Euclidean geometry to triangles big enough to represent areas swept out by planets in their orbits about the sun. Euclidean geometry, as an account of the geometry of physical space, is another classical commitment that relatively theory, in this case the General Theory of Relativity, has taught us to do without. I don't see this as sufficient reason to suppose that Kant would not have endorsed the present application of a Euclidean geometrical construction as an a priori demonstration of the point at issue in Newton's proof. Another question we must consider is whether we can deem this geometrical construction to have provided some appropriately strict universality.

Let us continue our examination of the proof. Note that the same argument we have explicated so far could be carried out for any number of additional instantaneous deflections that led to motions sweeping our equal areas in each equal unit of time. Note, also, that the argument will go through no matter how short the equal times between deflections are. This implies that the result will continue to hold in the limit as the times between deflections are made shorter and shorter; therefore, Newton's method of first and last ratios applies to convert this discrete result for instantaneous forces into the corresponding result for curves produced by continuously acting forces. I think Kant's discussion of continuous magnitudes in the Axioms of Intuition and the Anticipations of Perception shows that he would treat this part of the proof as an a priori demonstration as well. In an interesting paper on Kant's theory of geometry Michael Friedman makes a good case for the claim that such an underwriting of the applicability of differentiable magnitudes to phenomena was one of the main jobs Kant's account of constructions was designed to do (Friedman 1985, pp. 474-486).

The last step in the proof is Newton's appeal to Corollary V of the laws:

> The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion (Cajori 1962, p. 20).

This step extends the theorem to apply also to centers in uniform rectilinear motion. Newton appeals to his second law of motion to get this corollary. Friedman's discussion suggests that Kant would get this principle directly from his a priori commitment to the claims that absolute space and time are not possible objects of experience (Friedman 1986, p. 30).

Newton has more resources available than just this hypothetical derived from motions in a plane satisfying the law of areas by radii to a center to those motions due to a centripetal force, established in Theorem II. In Theorem I he had already proved the converse:

> The areas which revolving bodies describe by radii drawn to an immovable center of force do lie for the same immovable planes and are proportional to the times in which they are described (Cajori 1962, p. 40).

In Corollary VI he extends this to centers that move forward uniformly in right lines. These results are established under the assumption that the centripetal force at issue is the only force deflecting the body from uniform rectilinear motion. Otherwise, their proofs involve nothing beyond the assumptions I have discussed with respect to the proof of Theorem II. Thus, relative to this assumption about the existence of no other forces and the general background assumptions I have been discussing (the first law of motion, compositions of motions, Euclidean geometry, the calculus, the relativity of inertial motion), Newton has an *equivalence* between what he takes as Phenomenon V (the primary planets describe by radii drawn to the sun areas proportional to the times of description), and the theoretical claim he wants to establish (the primary planets are deflected from rectilinear motion into their orbits by centripetal forces directed to the sun).

The background assumptions and the equivalence they support allow for the possibility of alternative phenomena that would have been incompatible with the theoretical claim.[3] Indeed, Newton has available an additional corollary (Corollary I, Prop. II) that shows an equivalence between relevant values of the appropriate phenomenal magnitude (the rate at which the areas are swept out as increasing, constant, or decreasing with time), and relevant values of the theoretical magnitude of interest (direction of the deflecting force with respect to a radius to the center as advancing--off center in the direction of tangential motion; centripetal; or retarding--off center in the direction opposite to the tangential motion).[4] These results show that the background assumptions would, as it were, allow the value of the theoretical parameter to be read off from the value of the phenomenal parameter. They allow Newton to treat the motion of a body as if it

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3/ Thus, Newton's inference from this phenomenon meets the requirements for Clark Glymour's (1980) concept of bootstrap confirmation.

4/ Newton's inference meets requirements even stronger than Glymour's. It is typical of Newton's inferences from phenomena to be backed up by such an equivalence, which holds for a range of values the phenomenal magnitude might have taken.

were an instrument for detecting the direction of a deflecting force.

2. The Actual Phenomenon

On Kant's view, what is required to establish the law of areas for the primary planets as actual? One view would regard it as an empirical generalization established by induction from the relevant astronomical measurements. On this view the law would be supported by the same sort of inductive practice that underwrites curve fitting inferences generally. If this were all that were required, then the 'universal rule' I have just been discussing could be applied directly to the empirical law to establish the theoretical claim that the planets undergo centripetal forces. This would be a particularly simple picture of a way in which Kant's relatively rich a priori framework would allow theoretical claims that have the status of necessary truths (as explicated in the Third Postulate) to be established empirically.

Kant's discussion of the merely assumed and comparative universality that such empirical generalizations enjoy suggests that he would not regard induction (or any merely curve fitting inference) from the astronomical measurements as sufficient to establish the law of areas as actual. This is reinforced by the fact that we know, and Kant and Newton knew, that on the theory of universal gravitation the law of areas turns out to be false because of perturbations produced by planetary interactions. Such perturbations (that is, departures from the law of areas) for Saturn and Jupiter were actually observed by Flamsteed (after Newton urged him to look for them) some time between the first and second editions of the *Principia*.[5] Newton discusses these perturbations in Corollary III of Proposition V, Book III of that edition:

> Cor. III. All the planets do gravitate toward one another, by Cor. I and II. And hence it is that Jupiter and Saturn, when near their conjunction, by their mutual attractions sensibly disturb each others motions (Cajori 1962, p. 410).

5/ Newton first asked Flamsteed about this matter in a letter of 30 December 1684. He received an ambiguous response from Flamsteed dated 5 January 1685. The point came up, off and on, in their correspondence through the 1690s. The new corollary appears in a letter from Newton to Cotes dated 18 March 1712 (Hall & Tilling 1975).

However, Newton indicates no worry whatever about the effect of these perturbations on his argument for Proposition II.

Two relevant points strike the attentive reader of Newton's *Principia*. One is that the converse universal rule, Theorem I, establishes that motions satisfying the law of areas are the motions of an otherwise unperturbed body undergoing a centripetal force. This converse rule is subject to an auxiliary assumption that only the centripetal force is acting to deflect the body from uniform rectilinear motion. The interactions predicted by the theory of universal gravitation explicitly violate this assumption about initial conditions. This effectively prevents use of perturbations due to those interactions as evidence to establish the claim that the body is not undergoing a centripetal force.

Suppose one has established up to sufficiently fine tolerances the actual variation of the rate at which Jupiter sweeps out areas by radii to the sun as it moves in the plane of its orbit. Then the equivalence can be used to read off the direction of the *total* deflecting force at any instant. This will establish small departures from the central direction, advancing as Jupiter overtakes Saturn approaching conjunction, and retarding as it pulls away after passing conjunction. Similarly, Saturn's total deflecting force is respectively retarded and advanced for the moments when Jupiter is close behind or not far ahead. These small advances and retardations of the direction of the total deflecting force on each planet are, of course, exactly what result from composition of a basic centripetal force with attractions between them. The upshot of this first point is that, in the context of Newton's argument, such deviations are always taken together with other phenomena, corresponding to interactions, so that what gets derived is an instance where a centripetal force is composed with perturbing forces due to these interactions.

The second point that strikes an attentive reader of Proposition II, Book III (the proposition I am investigating):

> That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the sun; and are inversely as the squares of the distances of the planes of those planets from the sun's center (Cajori 1962, p. 406),

is that it claims that the centripetal forces retain the planets on their *proper* orbits. What is the proper orbit? I submit that according to Newton it is th: idealization of

Phenomenon V: The proper orbit is one that satisfies the law of areas. This is the orbit that would result under the influence of the centripetal force alone.

This suggests that Newton's phenomenon is a idealization guided by the very theory it is used to argue for, rather than any merely empirical generalization from the available astronomical measurements. Because the perturbations between Jupiter and Saturn are observable, the idealization actually conflicts with Flamsteed's new data. This changes the phenomenon's status from that of an idealization in the sense of a generalization from the data, to that of a more radical form of idealization which actually conflicts with some of the data. The first sort of idealization stays within whatever may be regarded as the admissible tolerances for measurement error.

In the first edition of the *Principia*, the theory of gravitation provided grounds for judging that the idealization of Phenomenon V was only an approximation to some more precise specification of the actual motions that would take into account interactions among the planets. But, until Flamsteed's measurements, both the idealized version and whatever corrected version might be forthcoming could be regarded as permitted by the empirical data. In contemporary curve fitting terms, we might say that the theory gave grounds for preferring some perturbed version to the smoother curve corresponding to the idealized phenomenon, even though both were actually within any established tolerances for measurement error.

Let us suppose that after Flamsteed's measurements the smoother curve was ruled out by new data points that fit the perturbations predicted by the theory. Newton, justly, regarded this as additional support for his theory. Our question was, why didn't he revise his derivation of Proposition II? The new sharper data make the derivation look like this. First, relatively simple, even radically idealized, assumptions about orbital phenomena are used to infer major inverse square centripetal forces. Then additional interaction forces are argued for, and these are then composed with the basic centripetal forces to generate new total deflecting forces that are then used, by means of Theorem I, to correct the idealizations about the phenomena. So long as the correction falls within the empirical tolerances, Newton could have taken it as his starting point. He would then derive the total deflecting force directly and apply the deflecting force derived from the interaction to arrive at the centripetal force. Newton doesn't need to change his starting point, so long as this constructive self-correcting procedure does end up predicting the new sharper data.[6]

Now we can say more about why Kant would not be satisfied with mere induction from the observations as a method for establishing to any very fine tolerances the actual variation of the rate at which Jupiter sweeps out areas to the sun in the plane of its orbit. What we need to establish is a family of relevant appearances sufficient to rule out any possible object of experience that would count as an instance in which the rate at which Jupiter sweeps out areas undergoes variation beyond the tolerances in question. What we have to start with is a set of observations of angles of Jupiter and the sun to the fixed stars from specified points on the earth at specified times. These are not perceptions of the relative positions of Jupiter with respect to the sun; however, Kant tells us

> The postulate bearing on the knowledge of things actual does not, indeed, demand immediate **perception** (and, therefore, sensation of which we are conscious) of the object whose existence is to be known. What we do, however, require is the connection of the object with some actual perception, in accordance with the analogies of experience which define all real connection in an experience in general (*KRV* A225/B272).

We need to connect the relative angles of the sun and Jupiter observed from our earth-bound reference frame to judgments about the relative position of Jupiter based on a reference frame fixed on the sun. This certainly requires appeal to kinematical and geometrical assumptions; but what does this have to do with the analogies? Presumably, we need to appeal to interactions in order to measure time differences at frames other than our own. For now, however, let us bypass this problem and allow that our set of observations somehow fixes a set of data points specifying, up to given tolerances, locations of Jupiter (in the plane of its orbit at various times) relative to the sun.

We still have the problem of generalizing from these data points. We need to be able to specify a host of other relevant appearances (relative positions at various times of Jupiter in its orbit) before we can pin down the actual variation of the rate at which Jupiter sweeps out areas by radii to the sun. I believe that Kant would hold that only a gener-

6/ Compare Ronald Laymon's (1983, pp. 192-196) interesting discussion of this point.

alization from the data that is also backed up by our best theory can count for us as an admissible specification of the actual rate.[7] The theory provides the connections according to the analogies between our data points and the host of other appearances we need to specify.

3. Material Necessity

I think Newton's achievement in his argument for universal gravitation was to formulate and to defend a new natural kind conception that would let us count motions of terrestrial bodies and motions of celestial bodies as phenomena of the same kind. The key step is the identification of the centripetal force on the moon with terrestrial gravity in Proposition IV. Newton's famous moon test shows that the inverse square centripetal force on the moon (calculated from its centripetal acceleration in its orbit) would, at the surface of the earth, equal the force of terrestrial gravity (established by Huygens' measurement of g, the acceleration of gravity, using a second's pendulum). He appeals to the following two rules to argue from the agreement with Huygens' measurement to the identification.

> Rule 1. We are to admit no more causes of natural things than are such as are both true and sufficient to explain their difference.

> Rule 2. Therefore, to the same natural effects we must, as far as possible, assign the same causes.

I think the role of these two rules is to function together to endorse something like the following unification principle for natural kind conceptions:

> Phenomena organized under two or more distinct natural kind conceptions ought to be brought under a single unified natural kind conception, insofar as this can be achieved.

As I see it the identification of the centripetal force on the moon with terrestrial gravity is crucial to a transformation of the conceptions of celestial and terrestrial mo-

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7/ History supports Kant here, for it turned out that Kepler's laws (even the law of areas) were not generally accepted in the scientific community until after they were backed up by (and, ironically, corrected by) Newton's theory (Wilson 1970).

tions that will make them count as phenomena of the same kind.

I have argued (Harper 1983) that this point of view illuminates some controversial features of Newton's argument. For example, some of the generalization steps, like the one extending the direct proportionality of gravitation to quantity of matter to all bodies, and like the one extending inverse square variation with distance to all gravitational attractions, seem to be legitimate only if they can be understood as natural kind inferences, analogues to our inference to extend to all samples of a kind of alloy the specific gravity we measure for a particular few given samples. On this point of view Newton's argument succeeds by establishing the force law of universal gravitation--between any two bodies there is a gravitational attraction directly proportional to their quantities of matter and inversely proportional to the square of the distance--as essential to his new conception for motions of bodies.

We can also learn something from Newton's example about what is needed to achieve a natural kind conception that unifies several ranges of phenomena. First, we have the requirement that the conception be able to predict the behavior of the phenomena from relevant initial conditions and that it continue to do so as approximations and simplifications are replaced by more realistic assumptions. Newton's example suggests that we also require deductions from phenomena in each of the ranges to be unified. He has the deductions of inverse square centripetal forces from orbital motions, and the deduction of direct proportionality to quantity of matter from terrestrial pendulum experiments and Jupiter's moons. Most saliently, he has the moon test! It offers distinct deductions of the value of g from phenomena from each of the two ranges to be unified. I think it plausible to suppose that as the web of such deductions from phenomena becomes more closely knit the kind conception becomes more unified.

Kant tells us that the unity of reason presupposes a certain idea of system for a whole of knowledge.

> This unity of reason always presupposes an idea, namely, that of the form of a whole of knowledge--a whole which is prior to the determinate knowledge of the parts and which contains the conditions that determine **a priori** for every part its position and relation to the other parts. This idea accordingly postulates a complete unity in the knowledge obtained by the understanding, by which this knowledge is to be not a mere contingent aggregate, but a system connected according to necessary laws (KRV A645/B673).

I submit that the fact that a kind conception itself must be appealed to in order to transform a phenomenon from a merely empirical generalization to an established actuality that can be used to infer material necessity according to the Third Postulate of Empirical Thought shows that it is functioning as a whole of knowledge that is prior to the determinate knowledge of its parts. I also think that the extent and interconnectedness of its web of such deductions from phenomena is a measure of the extent to which a natural kind conception approaches this ideal of complete systematization of the phenomena under it.

Kant tells us that reason prescribes and seeks to achieve systematization of the whole range of all our knowledge.

> If we consider in its whole range the knowledge obtained for us by the understanding, we find that what is peculiarly distinctive of reason in its attitude to this body of knowledge, is that it prescribes and seeks to achieve its **systematisation**, that is, to exhibit the connection of its parts in conformity with a single principle (KRV A645/B673).

He makes it clear that this idea, like other ideas of reason, is only a regulative ideal--a kind of imaginary point we can use to focus our scientific activity.

> This point is indeed a mere idea, a **focus imaginarius**, from which, since it lies quite outside the bounds of possible experience, the concepts of understanding do not in reality proceed; none the less it serves to give these concepts the greatest [possible] unity combined with the greatest [possible] extension (KRV 644/B672).

I think our principle that we should seek to unify natural kind conceptions whenever possible follows from taking this regulative ideal as a guide. Indeed, Kant formulates just such a unification principle as a logical maxim generated by the ideal.

> Now there is a logical maxim which requires that we should reduce, so far as may be possible, this seeming diversity, by comparing these with one another and detecting their hidden identity (A649/B677).

Kant refers to these hidden identities as powers (in a later passage on the same page). This certainly suggests a natural kind conception that specifies the essential nature (the power: for example, Newton's "there is a power of gravity pertaining to all bodies": Proposition VII), which explains and allows us to predict the phenomena organized under it.

I think one can view our scientific theories as taking over the roles of more primitive natural kind conceptions in organizing our understanding of and practical dealings with phenomena. Quine (1970) applauds as progress this replacement of more primitive tacit kind conceptions by more explicitly formulated scientific theories. He implies that one virtue of the scientific theories is that when they take over the role of guiding our understanding and practice, what he regards as the reprehensible modal commitments associated with the old kind conceptions can drop away as so much excess baggage. On my view, when the theories take over the roles of more primitive natural kind conceptions, they also take over some of the modal commitments Quine finds so reprehensible. When we come to regard a theory as explicating the true nature of some range of phenomena--Newton's theory of gravitation and laws of motion as explicating the nature of motions of bodies--we conceive it as having the validity of a necessary truth. It comes to explicate what we regard as a true constraint on what we can count as physically possible.

I think Kant's material necessity as explicated in the Third Postulate can be regarded as exactly this kind of physical necessity, a necessity that we attribute to those constraints we take to explicate the true nature of a natural kind. He tells us that we must postulate *for the object* the complete systematic unity demanded by our ideal of pure reason.

> The law of reason which requires us to seek for this unity, is a necessary law, since without it we should have no reason at all, and without reason no coherent employment of the understanding, and in the absence of this no sufficient criterion of empirical truth. In order, therefore, to secure an empirical criterion we have no option save to presuppose the systematic unity of nature as objectively valid and necessary (A651/B679).

On this view we must postulate or deem there to be physical constraints on possible objects of experience corresponding to the unreachable ideal of completely systematized and interconnected knowledge that reason demands we seek.

III. A Priori Knowledge

1. The A Priori and the Absolutely A Priori

I have explored the idea that when we come to regard a theory as explicating the true nature of a natural kind we conceive it as having the validity of a necessary truth in about the sense of "material necessity" Kant explicates in the Third Postulate. Now, to return to our earlier theme, Kant tell us that necessity is a mark of the a priori:

> First, then, if we have a proposition which in being thought is thought as necessary, it is an a **priori** judgment; and if besides it is not derived from any proposition except one which has the validity of a necessary judgment, it is an absolutely a **priori** judgment (KRV B3).

Is there a reasonable sense in which we can regard as a priori the status a theory achieves when it comes to be regarded as the explication of the essence in a natural kind conception? A brief look at the status accorded Newton's theory by 18th and 19th century figures suggests we may find something approximating Kant's weaker sense of 'a priori'.

When Newton couldn't account for the precession of the moon he put it down to a problem (the famous three body problem) of working out the difficult mathematical details of the required calculations. Clairaut, Euler and d'Alembert all tried unsuccessfully to solve this moon problem in the late 1740s, during a period when Newton's theory was still being assimilated on the Continent. d'Alembert, at one stage anyway, put the problem down to initial conditions--he investigated the idea that the moon might not be a sphere but a longer body we only see one end of--while Euler and Clairaut challenged the inverse square law (Waff 1976, pp. 50-83). In 1748 Clairaut re-analysed the mathematical details required for a Newtonian calculation. Perhaps his aim was to attempt a more conclusive refutation (Waff 1976, p. 175). But, he found that he was able to provide a Newtonian solution to the anomaly by taking into account components of the force of the sun on the moon (tangential to the earth-moon axis) that everyone had been ignoring (Waff 1976, p. 267ff.). Clairaut won a prize, and great renown, and the theory of gravitation was given a big boost on its way to becoming entrenched as one of our natural kind conceptions.

In the mid-19th century, when Adams found new difficulties with the moon's motions, there was no longer any question of tinkering with the inverse square law. The problem was put down to initial conditions: effects of the earth's tides on the moon. Even the difficulties with the perihelion of Mercury and the Michelson-Morley experiment were, as Kuhn (1962) puts it, treated as anomalies to be explained away. Only after Einstein's theory of relativity provided an alternative conception that could take over the role of Newton's theory for the organization and understanding of the phenomena of motion did these anomalies get transformed into empirical refutations.

One way in which we may regard a proposition as having a priori status as independent of experience is to put ourselves into an epistemic context with respect to it according to which we will not regard any merely empirical evidence as sufficient to refute it. This does not preclude an appeal to empirical evidence as part of what gets us into the special epistemic context in the first place, nor does it preclude that the context might change so that the proposition could loose its special status. This is what discovering or coming to understand a new conception that would provide sufficient organization of the relevant phenomena to count as a serious rival would do. It would change the context so that our proposition would no longer enjoy immunity from empirical refutation. I see no reason to suppose that 19th century physicists were not *warranted* in assigning such a status to Newton's theory. Moreover, such contextual a priori status seems to be a natural and reasonable aspect of a proper methodology for introducing and revising natural kind conceptions.

This is, at best, a pretty weak idea of a priori status. Kant's idea of the absolutely a priori seems to require more. When we look at Friedman's discussion of Kant's treatment of the laws of motion we begin to see promise of some appropriately stronger sense of the a priori:

> In other words, as in the modern "operational" approach, Kant views the Laws of Motion--in particular, the First and Third Laws--as definitive or constitutive of the spatio-temporal framework of Newtonian theory. These Laws must be presupposed in order first to define a notion of true (or actual) motion and thus the notion of a spatio-temporal framework (an inertial frame) within which to formulate further laws of nature: for example, the Law of Universal Gravitation. These fundamental Laws of Motion are therefore necessary to constitute the objective framework of Newtonian sci-

ence in the first place, and this, in the end, is why they are a priori for Kant (Friedman 1986, pp. 34-35).

Here we have the suggestion of an argument--a transcendental argument--to warrant assigning to some propositions the special status of being empirically irrefutable. There is an operationalist premise:

For Kant, Absolute Space and Absolute Time are not (even) possible objects of experience (Friedman 1986, p. 30);

and an account of how the laws of motion play a criteriological role in a constructive procedure for deriving the true motions from empirically accessible appearances (relative motions). This constructive procedure is an operational approximation to the unrealizable ideals of absolute space and time. Once we understand the criteriological role of the laws of motion we see that nothing could count as an empirical refutation of them.

Friedman's interpretive strategy adds to the sense in which we can claim a proposition to be immune to empirical revision by explicitly providing the warrant for this immunity. I think we can see, also, how the possibility of generating this warrant by coming to understand the transcendental argument adds another sort of independence from experience. It may well be that one could come to see the force of the argument for a special status for the laws of motion without any need to rely on empirical evidence to do so. If so, then we could say that the status of the laws is "absolutely independent of experience" in the sense that no merely empirical evidence is sufficient to refute the laws and also that there is no empirical evidence necessary to put one into the special context with respect to them. All of this, of course, doesn't show that such propositions (the laws of motion) cannot be revised. Once again, they can be dislodged by an alternative theoretical conception that is at least as adequate as they are for relevant explanatory purposes. In the present case, this would mean an alternative to the geometrical and kinematic assumptions built into the constructive procedure.

2. Geometry and Phoronomy

What about geometry? I think it's clear that Kant regards ostensive geometrical constructions as providing a priori warrant for taking the demonstrated theorems of Euclidean geometry as apodeictic constraints that any figures in space must satisfy (see Friedman 1985; Harper 1984). I also think that the space he intends is the three dimensional physical space in which we perceive things and move our bodies--this is the space of our form of outer intuition. Doesn't this commit me to the view that geometry is another area in which a new conception has overthrown the context within which our earlier demonstrations derived their warranting power? I don't think so!

I have argued (Harper 1984) that Kant's account of the warranting power provided by ostensive constructions in geometry depends on our capacity to survey the constructed figures and that this limits the precision to which they can warrant constraints on figures in space to tolerances corresponding to our perceptual competencies. This makes it possible to claim that the warranting power of such ostensive geometrical constructions is not undercut by the non-Euclidean geometries of contemporary physics, because the physical violations of the constraints occur on figures far beyond the scale at which we are competent to carry out our ostensive constructions. It also makes it possible to argue that the constraints ostensive constructions reveal are founded on physical features of our perceptual system and of the environment it has evolved, and that this system and its environment cannot be changed just by formulating new conceptions. If this is correct then the constraints provide a form of a priori warrant that is more independent of experience than those constraints we have already considered.[8]

There are two apparently devastating objections to any attempt to attribute such a view to Kant. First, he clearly wants to maintain that Newton was warranted in extending to very large triangles (for example, ones approximating areas swept out by Jupiter by radii to the sun) results he established by ostensive geometrical constructions on little diagrams like the ones I presented above. Second, there is a passage in which Kant tells us that the grossness of our senses does not in any way decide the form of possible experience in general.

Thus from the perception of the attracted iron filings we know of the existence of a magnetic matter pervading all bodies, although the constitution

8/ This a priori warrant may even approach the outrageously high standards set out by Philip Kitcher (1980) in his attempt to explicate a priori knowledge.

> of our organs cuts us off from all immediate perception of this medium. For in accordance with the laws of sensibility and the context of our perceptions, we should, were our senses more refined, come also in an experience upon the immediate empirical intuition of it. The grossness of our senses does not in any way decide the form of possible experience in general (KRV A226/B273).

The explicit point of this passage is, I think, that we should be willing to count non-observable states as actual if they are required by deductions from phenomena we do observe (for example, the behavior of the attracted iron filings); nevertheless, the claim at the end of the passage suggests that Kant would not regard surveyability of the large triangle as any requirement for Newton's warrant.

On my view Newton was indeed warranted in extending to such unsurveyably large triangles the result he obtained by ostensible construction on the small triangle, but this warrant involved a conception that is not demonstrable by any ostensive construction. This conception is the Euclidean ideal of homogeneity: that exactly the same geometrical constraints apply to figures of any size, no matter how large. For Kant, I think, the ideal of homogeneity results from an application of this idea as a regulative ideal which recommends that we be able to extend our geometrical framework to any system of possible objects of experience no matter how large that system may be.[9] As with other conceptions generated by regulative ideals, this conception can be undercut when we develop an alternative idealization that can take over its

9/ Homogeneity in this Euclidean sense may be seen as an instance of the more general requirement of regulative homogeneity which Kant takes to be an application of the logical law of reduction (KRV A649/ B677) I referred to above. It is a requirement involved in systematic unity, which is the central regulative need (KRV A656-664/ B684-692). Note also that in Kant's discussion of the principle of affinity ("a synthetic **a priori** proposition" of "objective but indeterminate validity", serving as a "rule for possible experience"), he extends the application of the law of gravitation (in ways that "experience can never confirm") to cover the hyperbolic paths of the comets, paths that move beyond our solar system, and, "passing from sun to sun", move to the most distant parts of the unlimited universe (KRV A662-663/B690-691). If the principle of affinity licenses inferences to items of such great size, there surely can be no problem involved in letting medium-sized constructions represent the very large triangles needed to develop proofs relying on the law of areas. These methodological considerations lend considerable support to my view of what counts as an a priori warrant.

function as a vehicle for approximating the unreachable goal the ideal demands that we seek. This is, of course, what actually happened when physics embraced non-Euclidean space.

Kinematics (Kant's phoronomy) is, of course, infected by any reconceptualization of the geometrical idealizations; but it has its own special revisable conception. Kinematics is not merely geometry, but an application of geometrical constructions under a new interpretation. This interpretation is a new conception, perhaps mostly pioneered by Galileo, under which lines and figures represent motions and compositions of motions. Built into this conception is the Galilean invariance principle to which Poincaré provided an alternative--an alternative that Einstein showed us we ought to adopt. Newton had no such alternative conception available; therefore, his warrant for his kinematical assumptions was a priori, exactly in the sense explicated by Kant in his account of phoronomy in MAN.[10]

10/ I want to thank Robert Butts for providing me with this opportunity to work out and express these views. The paper has benefited from his advice and encouragement over the years. He has been especially helpful in my attempts to formulate my views on unification. I also want to thank Graciela DePierris, Michael Friedman, and William Tait for a very helpful discussion of ostensive constructions in geometry. This material has also benefited from discussion with Kathleen Okruhlik, as have other parts of the paper as well.

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