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NOTES ON SEMANTICS

CONTENTS

SEMIOTIC

I. Semiotic and its parts

On *object language* (*L*) and *metalanguage* (*M*), see *Intr. Sem.*, sect. 1. An *expression* in *L* is a finite sequence of signs in *L* (*Intr. Sem.*, sect. 2). On sign-events (tokens) and sign-designs, expression-events, and expression-designs, see *Intr. Sem.*, sect. 3.

On the division of semiotic into three parts, viz., *pragmatics*, *semantics*, and *syntax*, see *Intr. Sem.*, sect. 4.

On the distinction between descriptive and pure syntax, see *Syntax*, sections 2 and 24, and *Intr. Sem.*, sect. 5. On descriptive and pure semantics, see *Intr. Sem.*, sect. 5. We shall here be concerned only with pure syntax and pure semantics.

II. Syntactical signs

Syntactical signs used in *M* as names (with numerical subscripts, e.g., 'A_1', 'A_2', etc.) or as variables (with letter subscripts, e.g., 'A_i', 'A_j', etc.) for expressions of the object language: '*A*' for expressions, '*s*' for signs, '*c*' for constants, '*v*' for variables, '*in*' for individual constants, '*inv*' for individual variables, '*pr*' for descriptive predicates, '*prv*' for predicate variables, '*S*' for sentential formulas (incl. sentences), '*D*' for designator formulas (incl. designators), '*K*' for classes of expressions (in most cases classes of sentences).

Thus, 'in_5' is short for 'the individual constant No. 5'. '$\sim pr_2$ (inv_5)' is short for 'that expression (of the object language) which consists of '$\sim$', followed by the predicate No. 2, followed by the left-hand parenthesis, followed by the individual variable No. 5, followed by the right-hand parenthesis'.

(I write 'iff' for 'if and only if'. I use as a sign of definition in M either 'iff or '$=_{\text{Df}}$'.)

A_i is an *open* expression $=_{\text{Df}}$ A_i contains a free variable.
A_i is a *closed* expression $=_{\text{Df}}$ A_i contains no free variale.

LOGICAL SYNTAX

III. Propositional calculus *PC*

A. Rules of formation

1. Signs of *PC*:
   (a) Constants '*B*', '*C*', etc.
   (b) Variables: '*p*', '*q*', etc.
   (c) Parentheses: '(', ')'.

2. Sentences of *PC*:
   (a) Any constant.
   (b) Any variable.
   (c) If S_i is a sentence, $\sim S_i$ is a sentence.
   (d) If S_i and S_j are sentences, $(S_i \vee S_j)$ is a sentence.

B. Rules of transformation.

1. Primitive sentences of *PC*:
   PS1. '$\sim(p \vee p) \vee p$'.
   PS2. '$\sim p \vee (p \vee q)$'.
   PS3. '$\sim(p \vee q) \vee (q \vee p)$'.
   PS4. '$\sim(\sim p \vee q) \vee (\sim(r \vee p) \vee (r \vee q))$'.

2. Rules of inference of *PC*:
   (a) Rule of substitution.
   (b) Rule of modus ponens.

2′. Formulation as definition:
   S_j is directly derivable in *PC* from K_i (or, from the sentences in K_i) iff either
   (a) for some S_i, K_i is $\{S_i\}$, and S_j is formed from S_i by substituting any sentence for a variable, or
   (b) for some S_i, $K_i = \{S_i, \sim S_i \vee S_j\}$.

IV. The calculus *PC′* (without variables)

A. Rules of formation.

1. Signs: (a), (c), (d) of *PC*.
2. Sentences: (a), (c), (d) of *PC*.

B. Rules of transformation.

1. Primitive sentence schemata:
   PS1. $\sim (S_i \vee S_i) \vee S_i$.
   PS2. $\sim S_i \vee (S_i \vee S_j)$.
   PS3. $\sim (S_i \vee S_j) \vee (S_j \vee S_i)$.
   PS4. $\sim (\sim S_i \vee S_j) \vee (\sim (S_k \vee S_i) \vee (S_k \vee S_j))$.
2. Rule of inference: modus ponens.

V. Definitions in general syntax, for any calculus *C*.

(1) R_k is a *proof* in $C =_{\mathrm{Df}} R_k$ is a finite sequence of sentences in *C* such that every sentence S_j of R_k is either a primitive sentence of *C* or directly derivable in *C* from a subclass of sentences which precede S_j in R_k.
(2) S_k is *provable* in $C =_{\mathrm{Df}} S_k$ is the last sentence of a proof in *C*.
(3) R_k is a *derivation with the premise-class* K_k in $C =_{\mathrm{Df}} R_k$ is a finite sequence of sentences in *C* such that every sentence S_j of R_k is either an element of K_k or a primitive sentence of *C* of directly derivable in *C* from a subclass K_i of the class of sentences which precede S_j in R_k.

(4) S_j is *derivable from* K_i in $C =_{\mathrm{Df}} S_j$ is the last sentence of a derivation with the premise-class K_i in C.

The following concepts are often useful in syntax and semantics.

(5) *The class* K *is closed* with respect to the relation R (or the function f) $=_{\mathrm{Df}}$ if $x_1, \ldots, x_n$ are elements of K *and* $R(y, x_1, \ldots, x_n)$ [or $f(x_1, \ldots, x_n) = y$, resp.], then y is an element of K. (Sometimes the following form is used: "if $K' \subset K$ and $R(y, K')$, then $y \varepsilon K$".)

(6) *The closure of the class* K with respect to the relations $R_1, \ldots, R_m$ (or the functions $f_1, \ldots, f_m$) $=_{\mathrm{Df}}$ the intersection of all classes which contain K as a subclass and which are closed with respect to $R_1, \ldots, R_m$ (or $f_1, \ldots, f_m$, resp.). (Tarski, 1941, sect. 47; Rosser, 1953, pp. 244 ff.)

We can then define (without the terms 'proof' and 'derivation'):

(7) The class of the *provable* sentences in $C =_{\mathrm{Df}}$ the closure of the class of primitive sentences with respect to direct derivability.

(8) The class of the sentences *derivable from* K_i in $C =_{\mathrm{Df}}$ the closure of the union of K_i with the class of primitive sentences with respect to direct derivability.

VI. Examples for *PC′* (see IV)

A. Example of a proof in *PC′*.

PS4	$\sim(\sim\overbrace{(B \vee B)}^{S_i} \vee \overbrace{B)}^{S_i} \vee (\sim\overbrace{(\sim B}^{S_k} \vee (B \vee B)) \vee (\sim B \vee B))$	(1)
PS1	$\sim(B \vee B) \vee B$	(2)
(1)(2)	$\sim(\sim B \vee (B \vee B)) \vee (\sim B \vee B))$	(3)
PS2	$\sim B \vee (B \vee B)$	(4)
(3)(4)	$\sim B \vee B$	(5)

B. Example of a derivation in *PC′*.

Premise	$C \vee B$	(1)
PS3	$\sim(C \vee B) \vee (B \vee C)$	(2)
(1) (2)	$B \vee C$	(3)

(For further examples of proofs and derivations, see Cooley, 1947, sections 29–31.)

VII. Rules of transformation of *PC*

Formulated with the *null class* Λ of sentences (*Intr. Sem.*, sect. 26 ff.)

(1) S_m is directly derivable (in *PC*) from $K_i =_{\text{Df}}$ one of the following conditions is fulfilled:
   (a) K_i is Λ, and S_m is $\sim(v_1 \vee v_1) \vee v_1$;
   (b) K_i is Λ, and S_m is $\sim v_1 \vee (v_1 \vee v_2)$;
   (c) K_i is Λ, and S_m is $\sim(v_1 \vee v_2) \vee (v_2 \vee v_1)$;
   (d) K_i is Λ, and S_m is $\sim(\sim v_1 \vee v_2) \vee (\sim(v_3 \vee v_1) \vee (v_3 \vee v_2))$;
   (e) (substitution, as in I);
   (f) (modus ponens, as in I).

(2) The class of sentences *derivable from* K_i in $PC =_{\text{Df}}$ the closure of Λ with respect to direct derivability.

(3) S_m is *provable* in $PC =_{\text{Df}} S_m$ is derivable from Λ.

SEMANTICS

VIII. Terminological remarks

A. We shall deal here with the *designative* (or cognitive) meaning component only, leaving aside all others (e.g., the emotive and the motivative meaning components). The designative meaning component is the one that is relevant for questions of truth. Thus our theory is *pure, designative semantics.* Therefore we consider only declarative sentences (called simply 'sentences') and their parts.

B. For the term '*designator*', see *M & N*, pp. 6 ff. We shall include among designators sentences, individuators (e.g., individual constants and individual descriptions), and predicators (e.g., predicates and lambda-expressions). All designators are closed expressions. Designators and open expressions of similar forms we call '*designator formulas*'.

C. *Terminology for kinds of expressions* (compare *M* & *N*, pp. 6 ff. and footnote 6).

	Closed Expressions	Constants	Variables
(1) designator formulas (incl. (2), (3), (4))	designators	designator constants	designator variables
(2) sentential formulas	sentences	sentential constants (or propositional constants)	sentential variables (or propositional variables)
(3) individual formulas	individuators	individual constants	individual variables
(4) predicator formulas	predicators	predicates	predicate variables

D. *Terminology of intensions and extensions.*

Designator	Intension	Extension
individuator	individual concept	individual
one-place predicator	property	class
n-place predicator	*n*-adic relation	class of *n*-tuples
sentence	proposition	truth-value (*T*, *F*; or 0, 1)

E. *Connectives and operators in M.*

In rules and technical statements in *M*, I shall sometimes write as follows:

1. *Parentheses* are used as in a symbolic language.
2. '*Not*' precedes its sentence; e.g., 'not (it rains)' for 'it does not rain'.

3. 'Or' is used in the non-exclusive sense; e.g., '*p* or *q*' for '*p* or *q* or (*p* and *q*)'.
4. I shall sometimes use symbolic *quantifiers* in *M*, e.g. '($\exists x$)' as short for 'there is an *x* such that'.
5. I shall sometimes use in *M* the *lambda-operator* for abstract-expressions, e.g., '(λx) (*x* is large)' as short for the property of being large' (or, in M^e, 'the class of those individuals which are large').
   The customary rule of *conversion* (Church) is used for these λ-expressions (see *M* & *N*, p. 3.)

IX. Semantical systems

A. A semantical system for an object language *L* contains at least rules of the following two kinds.
   (1) The *rules of formation* define 'sentence in *L*' (as in syntax).
   (2) *The rules of interpretation* give an interpretation for (i.e., specify the meanings of) all designators in *L*. These rules may have various forms. We shall use chiefly two forms;
      (a) rules of designation (Des^i) or intension,
      (b) rules of extension (Des^e) including rules of truth.

B. We distinguish *two operations* or investigations concerning any designator, e.g., a predicate pr_i or a sentence S_j (*M* & *N*, pp. 202 ff.):
   (1) The question of *meaning* or interpretation. In technical terms: "what is *the intension* of the designator?" The question is answered by an interpretation; technically, by the semantical rules of interpretation.
   (2) The question of factual *application*: e.g. "to which individuals does pr_i apply?", "is S_j true or false?". In technical terms, it is the question of the *extension* of the designator. The answer is (in general) found by an empirical investigation of facts.

X. The semantical system L_1

A.

Signs of L_1	Examples	Names in M
1) individual constants	'a_1', 'a_2', . . .	'in_1', 'in_2', . . .
2) one-place predicates	'P_1', 'P_2', . . .	'pr_1', pr_2', . . .
3) connectives	'$\sim$', '$\vee$'	
4) parentheses	'(', ')'	

L_1 contains no variables.

B. *Rules of Designation* (*Des*) *for* L_1. The relation *Des* holds in all and only those cases which are determined by the following rules:

R1. Individual constants.
(a) $Des(in_1$, Los Angeles),
(b) $Des(in_2$, the desk in Royce Hall 242), etc.

R2. Predicates.
(a) $Des(pr_1, (\lambda x)\,(x$ is large)),
(b) $Des(pr_2, (\lambda x)\,(x$ is red)), etc.

R3. Sentences.
(a) If $Des(pr_1, F)$ and $Des(in_j, x)$, then $Des(pr_i in_j, F(x))$.
(b) If $Des(S_i, p)$, then $Des(\sim S_i$, not $p)$.
(c) If $Des(S_i, p)$ and $Des(S_j, q)$, then $Des(S_j \vee S_j, p$ or $q)$.

(In the above rules, '*Des*' is used in three different types. An exact formulation which complies with the rule of types can be obtained either by attaching type indices to '*Des*' or by assigning '*Des*' to a transfinite level; see *Intr. Sem.*, p. 51.)

C. *Examples* of consequences from the *Des*-rules for L_1.
From R1(a), R2(a), and R3(a):

(1) $Des(pr_1 in_1, (\lambda x)\,(x$ is large) (Los Angeles),
hence by conversion:

(2) $Des(pr_1 in_1$, L.A. is large).
Further, again with conversion:

(3) $Des(\sim(pr_1 in_1 \vee \sim pr_2 in_2)$, not (L.A. is large or not (the desk R. H. 242 is red))).

The first argument expression in (3) is a spelling description (i.e., a description specifying each sign) for the sentence '$\sim(P_1a_1 \vee \sim P_2a_2)$' in L_1.

XI. Some definitions in general semantics for a semantical system L

A. Conditions of adequacy for designation in L.

A two-place predicate 'D' in M is an adequate predicate for designation in L only if the following condition is fulfilled:

For every designator A_i in L, a sentence in M of the form '$D(\ldots, ---)$', with a spelling description of A_i in the place of '...' and a translation of A into M in the place of '---', follows from the definition or the rules for 'D'. (*Intr. Sem.*, pp. 53 ff.)

On the basis of the rules in XB, '*Des*' fulfills this condition as an adequate predicate for designation in L_1; see the examples in XC.

B. General definitions of truth and falsity.

'*Des*' is supposed to be an adequate predicate for designation in L.

(1) A_i is *true* (in L) $=_{\text{Df}}$ there is a p such that $Des(A_i, p)$ and p.

(2) A_i is *false* (in L) $=_{\text{Df}}$ there is a p such that $Des(A_i, p)$ and not p.

Theorems.

(3) A_i is a sentence in L iff there is a p such that $Des(A_i, p)$.

(4) A_i is true in L or A_i is false in L, iff A_i is a sentence in L.

C. Example of a derivation in M for 'true in L_1'.

Premise	L.A. is large	(a)
rules XB for L_1	$Des(pr_1in_1$, L.A. is large)	(b)
(a), (b)	$Des(pr_1in_1$, L.A. is large) and L.A. is large	(c)
(c), exist. gen.	$(\exists p)\ Des(pr_1in_1, p)$ and p	(d)
(d), def. B(1)	pr_1in_1 is *true in* L_1	(e)

XII. Interchangeability in sentences with '*Des*'

A. *The three Des-relations.*

1. Suppose that '*Des*(. . ., – – –)' is directly obtained from the rules for '*Des* in *L*' and that (in accordance with XIA), '. . .' is a spelling description for a designator A_i in L and '– – –' is a translation of A_i into M. Under what condition shall we say that another sentence '*Des*(. . ., – . . –)', with another designator (in *M*) '– . . –' in the place of '– – –', holds likewise? The answer depends upon what is meant by '*Des*'.
2. We shall distinguish three semantical relations: Des^e, Des^i, and Des^s, characterized as follows. The derivative sentence holds
   (a) with 'Des^e', iff '– – –' and '– . . –' have the same extension,
   (b) with 'Des^i', iff '– – –' and '– . . –' have the same intension,
   (c) with 'Des^s', iff '– – –' and '– . . –' have the same sense.
3. Two designators are said to have (a) *the same extension* iff they are materially equivalent, and (b) *the same intension* iff they are logically equivalent (see *M & N*, sections 3 and 5).
4. We say that two designators have *the same sense* or are *synonymous* iff the one can be obtained from the other by transformations of the following kind:
   (a) replacement of a definiendum by its definiens or vice versa (or replacement of corresponding substitution instances);
   (b) rewriting of a bound variable;
   (c) lambda-conversion.

B. *Examples for* L_1.

In each of the subsequent examples (1) and (2), the sentence (a) is directly obtained from the rules XB for L_1 (in (2), (a) is supposed to be the rule for pr_5). Therefore, (a) holds for Des^e, Des^i, and Des^s. The second argument expression in (b) is synonymous with that in (a) (assuming suitable definitions for 'desk', 'featherless', and 'biped'). Therefore, (b) holds likewise for all three relations. The expression in (c) is logically equivalent but not synonymous with that in (a). Therefore, (c) holds for Des^e and Des^i, but not Des^s. Finally, the expression in (d) is materially equivalent to that in (a). Therefore, (d) holds for Des^e only.

	'Des^e'	'Des^i'	'Des^s'
1. *For a sentence in* L_1			
(a) '$Des(pr_1in_1 \vee pr_2in_2$, (λx) (x is large) (L.A.) or (λx) (x is red) (the desk . . .)),	*T*	*T*	*T*
(b) '$Des(pr_1in_1 \vee pr_2in_2$, L.A. is large or the writing table in R.H. 242 is red)'	*T*	*T*	*T*
(c) '$Des(pr_1in_1 \vee pr_2in_2$, the desk in R.H. 242 *is red or LA. is large*)'	*T*	*T*	*F*
(d) '$Des(pr_1in_1 \vee pr_2in_2$, Paris is in France)'	*T*	*F*	*F*
2. *For a predicate in* L_1			
(a) '$Des(pr_5$, (λx) (x is featherless and x is a biped)'	*T*	*T*	*T*
(b) '$Des(pr_5$, (λx) (x has no feathers and x has two feet)'	*T*	*T*	*T*
(c) '$Des(pr_5$, (λx) (x is a biped and x is featherless)'	*T*	*T*	*T*
(d) '$Des(pr_5$, (λx) (x is human)'	*T*	*F*	*F*

With 'Des^i' both in (1) and in (2), (a) is true but (d) is false, although the interchanged expressions have the same extension. Thus, sentences with 'Des^i' are bot extensional (*M & N*, sect. 11). The same holds for 'Des^s'. Therefore, 'Des^i' and 'Des^s' require non-extensional metalanguages. The extensional metalanguage (M^e) can accommodate only 'Des^e'.

XIII. Three metalanguages: M^e, M^i, M^s

A. Three identity signs.

We take M^e as a metalanguage containing 'Des^e': likewise M^i with 'Des^i', and M^s with 'Des^s'.

We shall use '$=^e$' in M^e as a sign of *identity of extensions;* likewise '$=^i$' in M^i for *identity of intensions,* and '$=^s$' in M^s for *identity of senses.* Thus, if '. . .' and '—' stand for designators (either sentences or predicators or individuators), then

(1) '$\ldots =^e$ - - -' is true in M^e iff '. . .' and '- - -' are materially equivalent;

(2) '$\ldots =^i$ - - -' is true in M^i iff '. . .' and '- - -' are logically equivalent;

(3) '$\ldots =^s$ - - -' is true in M^s iff '. . .' and '- - -' are synonymous.

(This use of '$=^s$' in M^s is like that of '$=$' in the symbolic object language in *M & N*, sect. 3).

B. Principles of interchangeability.

In all three cases, if '$\ldots =^{\circ\circ}$ – – –' holds in $M^{\circ\circ}$ (the superscript '$^{\circ\circ}$' stands for '*e*' or '*i*' or '*s*'), then '. . .' and '– – –' are interchangeable in any context, in accordance with the following principle:

(1) If A_i and A_j are designators in $M^{\circ\circ}$, $--A_i--$ is a designator containing A_i, and $--A_j--$ is formed from $--A_i--$ by replacing A_i by A_j, then from $A_i = A_j$, $--A_i-- =^{\circ\circ} --A_j--$ is deducible in $M^{\circ\circ}$.

Therefore:

(2) If $--A_i--$ and $--A_j--$ are sentences in $M^{\circ\circ}$, the following inference is valid in $M^{\circ\circ}$:

$$\frac{\begin{array}{l}A_i = A_j\\ --A_i--\end{array}}{--A_j--}$$

Hence, as a special case:

(3) The following inference is valid in $M^{\circ\circ}$:

$$\frac{\begin{array}{l}A_i =^{\circ\circ} A_j\\ Des^{\circ\circ}(\ldots, A_i)\end{array}}{Des^{\circ\circ}(\ldots, A_j)}$$

Thus the desired transformations of sentences with '$Des^{\circ\circ}$' (XII A2) are obtained.

C. General characterization of the three metalanguages for an object language L.

	M^e	M^i	M^s
1. *Designators in* $M^{\circ\circ}$ are interchangeable (a) iff they are	materially equivalent (in M^e)	logically equivalent (in M^i)	synonymous (in M^s)
(b) iff they have the same	extension	intension	sense
2. The values of the *variables in* $M^{\circ\circ}$ are	extensions	intensions	senses
3. The *semantics of L*, formulated in $M^{\circ\circ}$, can be based on a relation	*Des*^{e}	*Des*^{i}	*Des*^{s}
4. *Des*$^{\circ\circ}$ assigns to each *designator in L* an entity, namely	an extension	an intension	a sense
5. *Des*$^{\circ\circ}$ assigns the same entity to *two designators in L* iff they are	materially equivalent (in L)	logically equivalent (in L)	synonymous (in L)

D. Example for C5 in M^i.

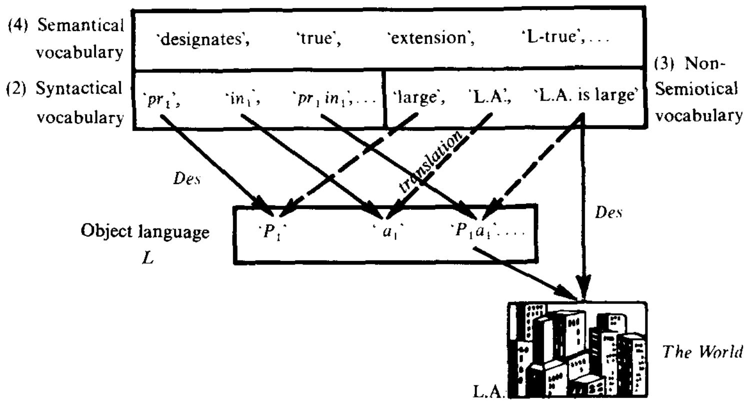
$pr_1 in_1 \vee pr_2 in_2$ (S_1) and $pr_2 in_2 \vee pr_1 in_1$ (S_2) are logically equivalent in L_1. From the rules of L_1 (XB):

Des^i (S_2, the desk . . . is red or L.A. is large).

By (XIIB) 1(C): *Des*^{i} (S_1, the desk . . . is red or L.A. is large). Thus *Des*^{i} assigns the same entity to S_1 and to S_2.

XIV. The vocabulary of the semantical metalanguage

A. Semantical metalanguage *M*.



B. The vocabulary of *the semantical metalanguage M* consists of the following four parts (the above diagram shows only some constants of the parts (2). (3). and (4)):

(1) *The logical vocabulary*: logical constants ('not', 'or', 'every', etc.) and general variables ('*x*', '*F*', '*p*', etc.).

(2) *The syntactical vocabulary*: Names of the signs in *L*, and a notation for concatenation. Thus a spelling description for any expression in *L* can be formulated. Further, syntactical variables (e.g., 'pr_i', 'A_j', etc.).

(3) *The non-semiotical vocabulary* (*translation vocabulary*): descriptive constants referring to non-linguistic entities (e.g., things in the world). This vocabulary, together with (1), must be sufficient for a translation of all sentences in *L*.

(4) *The semantical vocabulary.* The semantical terms are defined on the basis of the terms of the three other parts.

C. *The semantical theory* (pure semantics) contains only those sentences of *M* which

(a) contain at least one term of the semantical vocabulary (4), and

(b) are logically true.

(Thus the theory does not include '$pr_1 in_1$ is true in L_1'. This sentence is not a theorem of semantics but one of geography; it is factually true and logically equivalent to the sentence 'L.A. is large' in M and to '$P_1 a_1$' in L_1.

XV. The uniqueness of the designatum

A. The rules of $Des^{\circ\circ}$ *for* L_1 are as in XB. But now we can replace the formulation "*Des* holds in all *and only* these cases . . ." by a more exact one with '$x^{\circ\circ}$', in $M^{\circ\circ}$, adding the following rules to R1, R2, and R3.

R1 (α). For any in_i, x, and y, if $Des^{\circ\circ}$ (in_i, x), then $Des^{\circ\circ}$ (in_i, y) only if $x = {}^{\circ\circ}y$.

R2 (α). For any pr_i, F, and G, if $Des^{\circ\circ}$ (pr_i, F), then $Des^{\circ\circ}$ (pr_i, G) only if $F = {}^{\circ\circ}G$.

R3 (α). For any A_i,, p, and q, if $Des^{\circ\circ}$ (A_i, p) then $Des^{\circ\circ}$ (A_i, q) only if $p \doteq {}^{\circ\circ}q$.

B. Theorems, based on the above rule for L_1.

(1) For any in_i, x, and y, if $Des^{\circ\circ}$ (in_i, x), then $Des^{\circ\circ}$ (in_i, y) *iff* $x = {}^{\circ\circ}y$.

(2) For any pr_i, F, and G, if $Des^{\circ\circ}$ (pr_i, F), then $Des^{\circ\circ}$ (pr_i, G) *iff* $F = {}^{\circ\circ}G$.

(3) For any A_1, p, and q, if $Des^{\circ\circ}$ (A_i, p), then $Des^{\circ\circ}$ (A_i, q) *iff* $p = {}^{\circ\circ}q$.

These theorems together with the rules XB say that, for each designator in L_1, there is exactly one designatum$^{\circ\circ}$ (i.e., entity assigned to it by $Des^{\circ\circ}$).

C. Sufficient and necessary condition of adequacy for designation in L.

A two-place predicate 'D' in $M^{\circ\circ}$ is an adequate predicate for designation$^{\circ\circ}$ in L *iff* the following two conditions are fulfilled:

(1) the condition in XIA (which is only a necessary condition of adequacy),

(2) the condition of the *uniqueness of the designatum*$^{\circ\circ}$: Every

sentence in $M^{\circ\circ}$ of the form 'if $D(\ldots, ---)$ and $D(\ldots, -.-)$, then '$--- =^{\circ\circ} -.-$' follows from the definition or the rules for 'D'.

On the basis of the theorems under (B), $Des^{\circ\circ}$ for L_1 fulfills also the condition (2) and therefore is an adequate predicate in $M^{\circ\circ}$ for designation$^{\circ\circ}$ in L_1.

D. Theorems.

Henceforth, when we refer to a relation $Des^{\circ\circ}$ for a system L it is assumed that the rules for it in $M^{\circ\circ}$ are such that the adequacy condition (C) is fulfilled. Then the following holds for L.

(1) For any A. and p, if $Des^{\circ\circ}$ (A_i, p), then
 (a) A_i is true iff p;
 (b) A_i is false iff not p.

(2) For any A_i, if A_i is true, A_i is not false. (Indirect proof. The assumption that A_i is both true and false leads to the conclusion that, for some p, p and not p, which is impossible.)

(3) For any sentence S_i, S_i is false iff S_i is not true.

XVI. Truth

A. Rules of truth for L_1.

The following rules may take the place of the rules of $Des^{\circ\circ}$ for sentences (XB, R3 and XVA) and the definition of truth (XI B (1)); they lead to the same results concerning truth in L_1.)

R3′.

(a) If $Des^{\circ\circ}(pr_i, F)$ and $Des^{\circ\circ}(in_j, x)$ then $pr_i in_j$ is true (in L_1) iff $F(x)$.

(b) $\sim S_i$ is true iff S_i is not true.

(c) $S_i \vee S_j$ is true iff S_i is true or S_j is true.

The usual *truth-tables* for sentential connectives are diagram formulations of rules of truth, corresponding to formulations in words like (b) and (c) above.

The rules R3′ give a recursive definition for 'true in L_1'. (An equivalent explicit definition can be formulated with the help of the concept of closure, V (6).)

B. Sufficient and necessary condition of adequacy for truth in a system L.

A one-place predicate 'T' in M is an adequate predicate for truth in L iff the following condition is fulfilled. For every sentence S_i in L, a sentence in M of the form '$T(\ldots)$ iff $---$; with a spelling description of S_i in the place of '...' and a translation of S_i into M in the place of '$---$', follows from the definition of the rules for 'T'. (This is the so-called Leśniewski requirement; see Tarski, "Wahrheitsbegriff", p. 305, "Konvention W"; Tarski, "The semantic conception of truth", sect. 4; Carnap, *Intr. Sem.*, pp. 26–29.)

Suppose that 'T' fulfills the above condition. Then '$T(\ldots)$ iff $---$' is logically true. Therefore, the sentence '$T(\ldots)$' in M is logically equivalent to the translation of S_i into M and hence also to S_i.

Thus, to assert that a sentence is true means the same as to assert the sentence itself (see Tarski, "The semantic conception of truth", and Carnap, "Remarks on induction and truth", sect. 3). If 'true in L_1' is introduced in either of the two ways mentioned in (A), then the condition of adequacy is fulfilled.

XVII. Denotation

A. Following R.M. Martin (*J.S.L* **18**, 1953, 1–8), we use '*denotes*' in such a sense that 'P_1' in L_1 is said to denote every single individual which is large (not, as in traditional terminology, the class of large individuals). In $M^{\circ\circ}$, we can define '$Den^{\circ\circ}$' on the basis of '$Des^{\circ\circ}$':

(1) $Den^{\circ\circ}(A_i, x) = {}^{\circ\circ}(\exists F)\,[Des^{\circ\circ}(A_i, F) \text{ and } Fx]$.

On the other hand, if '$Den^{\circ\circ}$' is introduced by $Den^{\circ\circ}$-rules (see below), we can define '$Des^{\circ\circ}$' for predicators:

(2) $Des^{\circ\circ}(A_i, F) = {}^{\circ\circ}(F = {}^{\circ\circ}(\lambda x)[Den^{\circ\circ}(A_i, x)])$.

Hence we obtain:

(3) For any predicator A_i, $Des^{\circ\circ}(A_i, (\lambda x)\,(Den^{\circ\circ}(A_i, x)))$.

[The above definitions and the subsequent $Den^{\circ\circ}$-rules may be used in M^e and M^i. It is, however, doubtful whether the adherents of a sense-logic would admit them in M^s.]

B. Rules of denotation. If *Des*-rules for predicates are used, predicate variables are needed in the formulation of *Des*-rules for sentences (e.g., in XB, R3(a)) or of truth-rules (e.g., in XVIV, R3′ (a)). If, instead, *Den*-rules for predicates are used, predicate variables are not needed (unless the object language contains predicate variables). [However, predicate variables are needed for an explicit definition of truth with the help of closure.] Furthermore, *Den*-rules do not contain λ-expressions. We can then obtain results of the simple form (d) in the examples in (D) and (E) below without the use of λ-conversion (compare XC(2)). We may then drop λ-conversion in the definition of synonymy (XIIA4(c)), which seems preferable.

C. *Den*-rules for L_1. The following rules may take the place of XB, R2 and XVA, R2(α):

R2′ *Den*-rules for predicates in L_1.
(a) $Den^{\circ\circ}(pr_1, x) = {}^{\circ\circ}(x \text{ is large})$.
(b) $Den^{\circ\circ}(pr_2, x) = {}^{\circ\circ}(x \text{ is red})$.
etc.

Then XVIA, R3′(α) is replaced by the following rule:
R3″ (α). If $Des^{\circ\circ}(in_j, x)$ then $pr_i in_j$ is true (in L_1) iff $Den^{\circ\circ}(pr_i, x)$.

D. Example of a proof, with the rules in (C).

XB, R1(α)	$Des^{\circ\circ}(in_1, \text{L.A.})$	(a)
R2′(α)	$Den^{\circ\circ}(pr_1, \text{L.A.}) = {}^{\circ\circ}(\text{L.A. is large})$	(b)
(a), R3″(α)	$pr_1 in_1$ is true in L_1 iff $Den^{\circ\circ}(pr_1, \text{L.A.})$	(c)
(b), (c), XIIIB(2)	$pr_1 in_1$ is true in L_1 iff L.A. is large	(d)

E. *Des*-rule for atomic sentences in L_1 based on the *Den*-rules. On the basis of the rules R2′ above, the following rule takes the place of XB, R3(α), while (b) and (c) remain unchanged.
R3‴(α). If $Des^{\circ\circ}(in_j, x)$, then $Des^{\circ\circ}\,(pr_i\ in_j\ Den^{\circ\circ}(pr_i, x))$.

Example of a proof.
(a) and (b) as in (D).

(a), R3‴(α) $Des^{\circ\circ}(pr_1 in_1, Den^{\circ\circ}(pr_1, \text{L.A.}))$ (c)
(b), (c), XIIIB(3) $Des^{\circ\circ}(pr_1 in_1, \text{L.A. is large})$ (d)

XVIII. Interpretation in the extensional metalanguage M^e

A. Do the rules for 'Des^e in L' convey the intended interpretation of L?

The rules are supposed to fulfill the condition of adequacy (XVC). Hence, for any designator A_i, the rules yield a Des^e-sentence containing a translation of A_i. E.g., for L_1, we obtain:
(1) $Des^e(pr_1 in_1, \text{L.A. is large})$.
But the following sentence is likewise true:
(2) $Des^e(pr^1 in^1, \text{Paris is in France})$.
This shows again that Des^e is the relation between a designator and its extension, *not its meaning.* The sentence (1) does convey information about the intended meaning of $pr_1 in_1$, but (2) does not. (1) follows logically from the rules, while for (2) the factual premise 'L.A. is large $=^e$ Paris is in France' is needed. In every sentence of the form '$Des^e(pr_1 in_1, ---)$', which follows logically from the rules, '$---$' is logically equivalent to 'L.A. is large'. Therefore every sentence of this kind gives the intended interpretation (logical meaning, content) of $pr_1 in_1$. In this sense, the intended interpretation of L is conveyed by the Des^e-rules in M^e.

B. Do the rules for 'true in L', formulated in M^e, convey the intended interpretation of L?
The rules are supposed to fulfill the condition of adequacy (XVIB). The situation is analogous to that in (A). It is often said that to understand the meaning of a sentence is to know under what conditions it would be true. But this should be qualified as follows: a *logically true* statement of a (necessary and sufficient) truth condition for a sentence S_i conveys the meaning of S_i. For example, (3) and (4), are both true in M^e.
(3) $pr_1 in_1$ is true in L_1 iff L.A. is large.

(4) $pr_1 in_1$ is true in L_1 iff Paris is in France.

(3) gives the intended meaning, (4) does not. In every sentence in M^e of the form '$pr_1 in_1$ is true in L_1 iff – – –' which follows logically from the rules, '– – –' is logically equivalent to 'L.A. is large'. Thus the rules in M^e for 'true in L' convey the intended interpretation of L.

XIX. Philosophical issues concerning the semantical concept of truth

A. Some philosophers commit a *confusion of 'true' with 'verified'* (or 'known to be true', 'well established', 'highly probable', etc.). The distinction is important. Verification is relative to person and time, truth is not. Read pp. 119–123 of Carnap, "Truth and Confirmation", in Feigl-Sellars.

For (B) to (F), *read* Tarski, "The semantic conception of truth", esp. Part II "Polemical remarks".

B. *Objection*: 'true' can be eliminated, and thus is *useless.* (See Tarski, sect. 16, 20–22.)

C. The question of agreement with the *classical conception* (correspondence theory of truth, e.g. Aristotle). (See Tarski, sect. 17.)

D. The question of agreement with *every-day usage.* (See Tarski, sect. 17; compare Ness.)

E. *Objection*: the concept has *no philosophical importance.* (See Tarski, sect. 18.) Later Black raised this objection ("The semantic definition of truth"); for his main arguments, see (G), (H), (J) and (K) below.

F. *Objection*: the semantical conception of truth *involves metaphysics.* (See Tarski, sect. 19, with reference to Nagel; compare Nagel, 1944, p. 67n.)

G. *Objection*: the concept is defined only for artificial languages. Black (sect. 6) believes that only a definition for colloquial English would be philosophically relevant (sect. 6). He admits that, in principle, such a definition could be given; but then objection (H) would hold.

H. *Objection*: The definition of truth is not general, but is based on an enumeration of instances; any attempt to generalize the set of sentences of the form '... is true iff – – –' referred to in the adequacy condition (XVI B) leads to nonsensical formulations: we seem to understand the general principle underlying the definition, but this principle cannot be formulated (Black, sect. 3, 4, 6, 7). *Answer.* The definition can be stated in a general form in terms of designation (see XI B (1)); this was D17-C1 in *Intr. Sem.*). This form of the definition expresses the underlying principle. The definition is based on a general definition for '*Des*' with respect to sentences (X B R3). This, in turn, is based on *Des*–rules for individual constants and predicates (X B, R1, R2). The latter rules proceed indeed by enumeration; this is inevitable because the interpretation of a language must ultimately go back to its dictionary.

J. *Objection*: The semantical definition of truth is *neutral with respect to the philosophical controversy*; the adherents of the correspondence theory, the coherence theory, and the pragmatist theory of truth would all accept the sentences of the form '... is true iff – – –' specified in the adequacy condition (XVI B) (Black, sect. 8, 9). *Answer.* If this were the case, then the three theories would be based on essentially the same concept, because if each of two predicates fulfills the adequacy condition, then they are logically equivalent.

K. *Objection*: The philosophically important concept of truth is *not*, like the semantical concept, *a property of sentences* expressed in the metalanguage, but rather a concept used in the object language in the form "it is true that ...". (Black, sect. 8; Strawson, "Truth", *Analysis* 9, 1949, reprinted in Macdonald: "Truth is not a property of symbols; for it is not a property.") *Answer.* This use of 'true' seems indeed more frequent in the everyday language. It is useful for purposes of emphasis, opposition, and the like. But its usefulness for theoretical purposes, i.e., for expressing cognitive content, is nil. It can be explicated by the explicit definition:

(1) $T(p) =_{\mathrm{Df}} p$ (*Intr. Sem.* D17–1).
And analogously for "It is false that . . .":
(2) $F(p) =_{\mathrm{Df}} \sim p$.
Thus '*T*' and '*F*' are extensional connectives. '*F*' is merely a sign of negation. '*T*' is the redundant connective; its omission does not change the content. E.g., the following two sentences are logically equivalent:
(3) It is true that L.A. is large.
(4) L.A. in large.
Likewise the following two:
(5) It is false that L.S. is large.
(6) L.A. is not large.

XX. Semantical system for the language L_2 with individual variables

A. General remarks on a language *L*.

1. Let *L* be an object language with individual variables. Rules of interpretation for *L* must contain a rule specifying the *domain of individuals* of *L*, i.e., the class of those entities which are to be taken as the values of the individual variables. The domain may be infinite. It is not required that *L* contain an individual constant for every individual in *L*.
2. A *value assignment* (*VA*) for the individual variables in *L* is a function which assigns to every individual variable in *L* one individual. We take in *M* 'VA_1', 'VA_2', *etc.* as constants for *VA*, and 'VA_k', 'VA_m', etc. as variables. We write, e.g., '$VA_2(inv_4)$' as short for 'the individual assigned by VA_2 to inv_4'.
3. An open designator formula, e.g., '$P_1x_1 \vee P_2x_3$', does by itself not designate anything. However, we can give an interpretation for it by specifying what the formula

designates with respect to a given VA_k; $VA_k(inv_1)$ is then, so to speak, taken as the designatum of inv_1. Note that a *VA* assigns an individual, not an individual constant. This has the advantage that open formulas can be interpreted even if *L* does not contain individual constants for all individuals.

4. In the following our metalanguage will always be M^e, unless the contrary is stated. We write briefly '*M*' for 'M^e', '=' for '$=^e$', '*Des*' for 'Des^e', and '*des*' for 'des^e'. Because of the uniqueness of the designatum (XV), we may use a *functor* '*des*' instead of the predicate '*Des*'. We write '$des(A_i)$' for 'the designatum of A_i', i.e., 'the entity (extension) to which A_i bears the relation *Des*'. We write '$des_k(A_i)$' for 'the designatum (i.e., the extension) of A_i with respect to the value assignment VA_k'.

B. Signs of L_2.

(1) – (4) like those of L_1 (XA).
(5) Individual variables, e.g. 'x_1', 'x_2', etc. Their names in *M*: 'inv_1', 'inv_2', etc.

C. Rules of formation for L_2.

An expression A_i in L_2 is a *sentential formula* in L_2 iff A_i has one of the following five forms, where S_j and S_k are sentential formulas:

(1) $pr_i in_j$ (atomic sentence),
(2) $pr_i inv_j$ (open atomic sentential formula),
(3) $\sim S_j$ (negation),
(4) $(S_j \vee S_k)$ (disjunction),
(5) $(inv_i)(S_j)$ (universal sentential formula).

A_i is a *sentence* in $L_2 =_{Df} A_i$ is a closed sentential formula in L_2.

D. Rules of interpretation for L_2.

RI The *individuals* in L_2 are the material bodies at a given time t_0.

The subsequent rules RD 1 to 6 constitute a recursive definition which enables us to determine $des_k(A_i)$ for any designator formula A_i with respect to any VA_k. Later the function *des* will be defined, but only for designators, i.e., closed designator formulas. For any sentential formula S_i, $des_k(S_j)$ is one of the truth-values, either T (for truth) or non-T (for falsity), ('T' is a sentential constant in M; it may be regarded as short for, say, '$a_1 = a_1$'.) If a rule says: "$des_k(S_i) = T$ iff . . .", this is meant to imply that, if the condition ". . ." is not fulfilled, $des_k(S_i) =$ non-T. "$des_k(S_1) = T$" says in effect that S_1 is true with respect to VA_k or that S_1 is satisfied by VA_k. Thus the rules RD yield, for any sentence S_i in L_2, a necessary and sufficient condition for the truth of S_i with respect to any given VA_k. While RD 1 and 2 are analogous to X BR 1 and 2 for L_1, RD 4 to 6 are analogous to the truth-rules XVI R3′ (a) to (c).

RD. *Rules of des_k for L_2.*

RD1. For predicates.
(a) $des_k(pr_1) = (\lambda x)$ (x is large),
etc.

RD2. For individual constants.
(a) $des_k(in_1) =$ Los Angeles,
etc.

RD3. For individual variables. For any inv_j, $des_k(inv_j) = VA_k(inv_j)$.

RD4. If s_j is either an individual constant or an individual variable and $des_k(s_j) = x$ and $des_k(pr_i) = F$, then $des_k(pr_i s_j) = T$ iff $F(x)$.

RD5. $des_k(\sim S_i) = T$ iff $des_k(S_i) =$ non-T.

RD6. $des_k(S_i \vee S_j) = T$ iff $des_k(S_i) = T$ or $des_k(S_j) = T$.

RD7. Let S_i be $(inv_i)(S_j)$. Then $des_k(S_i) = T$ iff, for every value assignment VA_m that differs from VA_k at most for inv_i, $des_m(S_j) = T$.

E. Example for *RD*7.

(Here and in the examples G, we shall use 'S_1', 'S_2', etc. also as individual constants in *M*.) Let S_2 be a universal sentential formula of the following form with free variables 'x_2' and 'x_3'; let the operand be S_1.

$$S_2: \qquad (x_1)\overbrace{(\ldots x_1 \ldots x_2 \ldots x_3 \ldots)}^{S_1},$$

	x_1	x_2	x_3	
Individuals assigned by VA_k:	a_2	a_3	a_5	
Individuals assigned by VA_{m_1}:	a_1	a_3	a_5	
Individuals assigned by VA_{m_2};	a_2	a_3	a_5	$(= VA_k)$
Individuals assigned by VA_{m_3}:	a_3	a_3	a_5	
etc.	etc.			

VA_{m_1}, VA_{m_2}, etc. are those *VA* which differ from the given VA_k either for no variable (e.g., VA_{m_2}) or for 'x_1' only. Thus, according to RD7, S_2 is true with respect to the given VA_k iff S_1 is true for those values of 'x_2' and 'x_3' which are assigned by VA_k and for *every* value of 'x_1'.

F. Definitions of designation, truth, and falsity. According to the rules RD 1 to 7, if A_i is a designator (and hence closed), then $des_k(A_i)$ is independent of VA_k. Therefore we define as follows, using an arbitrarily chosen VA_1:

(1) For any designator A_i in L_2, $des(A_i) =_{\text{Df}} des_1(A_i)$, where VA_1 assigns a_1 to every individual variable.

(2) A_i is *true* in $L_2 =_{\text{Df}} A_i$ is a sentence and $des(A_i) = T$.

(3) A_i is *false* in $L_2 =_{\text{Df}} A_i$ is a sentence and $des(A_i) =$ non-T.

G. Examples.

1. For an atomic sentence. Let S_3 be $pr_1 in_1$. For any VA_k, hence also for VA_1, we obtain from RD 1, 2, 4 (writing 'a_1' for 'Los Angeles'):

   (a) $des_1(S_3) = T$ iff a_1 is large.

   By F(1), and with ' = ' for 'iff' (see XIII A(1)):

   (b) $(des(S_3) = \text{T}) = a_1$ is large.

   The following is a tautology (see, e.g., *Foundations*, T21–5u(1)):

(c) $(p = T) = p$.
Hence from (b):
(d) $des(S_3) = a_1$ is large.
This result corresponds to XC(2) for L_1. From (b), with F(2):
(e) S_3 is true iff a_1 is large.

2. For a universal sentence. Let S_1 be 'P_1x_1' and S_2–'$(x_1)(P_1x_1)$'. Take the various *VA*'s as in the earlier example E, but now with 'x_1' as the only free variable in S_1. Then $VA_k(inv_1) = a_2$, and, for every i, $VA_{m_i}(inv_1) = a_i$. Hence by RD 3, $des_k(inv_1) = a_2$ and, for every i, $des_{m_i}(inv_1) = a_i$. by RD 1(a) and RD 4:
(a) For every i, $des_{m_i}(S_1) = T$ iff a_i is large.
Hence by RD 7:
(b) $des_k(S_2) = T$ iff, for every VA_{m_i}, $des_{m_i}(S_1) = T$.
(c) $des_k(S_2) = T$ iff, for every i, a_i is large.
(d) $des_k(S_2) = T$ iff every individual is large.
Since S_2 is closed, (d) holds for every VA_k, hence also for VA_1.
Then by F(1):
(e) $des(S_2) = T$ iff every individual is large.
Hence with (c) of Example (1):
(f) $des(S_2) =$ every individual is large.
From (e) with F(2):
(g) S_2 is true iff every individual is large.
Since 'every individual is large' is the translation of S_2, the adequacy conditions for designation and for truth are fulfilled.

XXI. Preliminary explanations of the language L_3 with a type system

A. The metalanguage *M*.

1. We use, as before (XX A4), *M* (i.e., M^e) and we write '=' and '*des*'.

2. We use in M numerals '1', '2', etc. as individual constants. They refer to enumerated *positions* in an ordered domain. (Thus M is a coordinate language, see *M & N*, sect. 18.) '3 is blue' is understood as 'the position No. 3 is blue'. We use as individual variables in M '*j*', '*k*', . . . , '*n*'. Their values are numbers ('number' is here meant as 'positive integer') and, secondarily, positions (see *M & N*, p. 86).
3. We sometimes use in M '*u*', '*v*', etc. as variables without fixed type. (Strictly speaking, they should be regarded as variables of a transfinite level, see the remark in X B.)

B. The system of types. This system serves as a classification of designator formulas of the object language L_3, and also as a classification of the corresponding extensions. The types are 0, 1, 2, etc. Type 0 comprises the individuals, type 1 the classes of individuals, type 2 the classes of classes of individuals, etc. The *m*-the constant of type n ($n = 0, 1, 2, \dots$) in L_3 consists of the letter '*a*' followed by n (0 or more) superscript primes, followed by m (one or more) subscript primes. As a convenient unofficial notation, we write numerals instead of the strings of primes, e.g., 'a_3^2' and 'a_4^0'. The *m*-th variable of type n consists of '*x*' with n superscript primes and m subscript primes. The names in M of constants in L_3 are formed with '*c*', and the names of variables with '*v*'; e.g., 'c_3^2' is the name of 'a_3^2', and 'v_4^0' is the name of 'x_4^0'. We use '*n*' and '*m*' as numerical variables in M.

Classification of designator formulas:

Class	Designator Formulas		Extensions
	Open or closed	Closed	
S	sentential formulas	sentences	truth-values
type 0	individual formulas	individuators	individuals
type 1			
type 2	predicator formulas	predicators	Classes
etc.			

C. Other signs and expressions in L_3.

1. L_3 contains connectives for negation and disjunction. '$\equiv$' is used as sign of identity of extensions (like '$=^e$' in M^e, XII A), hence also as a biconditional connective (*M* & *N*, pp. 13 f.).
2. Variables of all types are used in L_3 in *three kinds of operators:*
   (a) universal quantifiers: (v^n_m);
   (b) lambda-operators for class expressions (*M* & *N*, p. 3): (λv^n_m);
   (c) iota operators for descriptions: $(\imath v^n_m)$.
3. In order to assure that a closed *iota-description* D_j of the form $(\imath v^n_m)(S_k)$ has a unique designatum even if S_k does not fulfill the uniqueness condition (i.e., if either no extension of type n or more than one satisfy S_k), we make the following convention (see *M* & *N*, sect. 8, method III b):

	Type n	$des(D_j)$ in the case of non-uniqueness
(a)	0	an arbitrarily chosen individual, say, the position No. 1
(b)	$n\,(>0)$	the null class of type n

The rules for des_{pr} and des for L_3 (in XXIV and XXVI) will be made so as to yield these results.

XXII. Rules of formation for L_3

A. L_3 contains the following eleven *signs:*

	Sign in L_3	Explicit name of sign in *M*
(1)	$\sim$	sign of negation
(2)	$\vee$	sign of disjunction
(3)	$\equiv$	sign of identity (and biconditional)
(4)	a	sign of constants
(5)	x	sign of variables

	Sign in L_3	Explicit name of sign in M
(6)	λ	lambda
(7)	ι	iota
(8)	(	left-hand parenthesis
(9)	)	right-hand parenthesis
(10)	′	superscript prime
(11)	′	subscript prime

In spelling descriptions in M, we use 'c' as the name of 'a', 'v' as the name of 'x', and each of the other signs as a name of itself. For convenience, we take 'c_3^2' as the name of 'a_3^2', and 'v_1^0' as the name of 'x_1^0' etc.

B. Designator formulas in L_3.

An expression is a designator formula in L_3 iff it has one of the following ten forms, where D_j^n and D_k^n are any *designator formulas* of type n, and S_j and S_k are any *sentential formulas*, i.e., formulas of class S. (For 'designator' and other terms, see VIII C.)

	Form of Expression	Class
(1)	c_m^n	type n
(2)	v_m^n	type n
(3)	$D_j^{n+1}\,(D_k^n)$	S
(4)	$(D_j^n \equiv D_k^n)$	S
(5)	$(S_j \equiv S_k)$	S
(6)	$\sim S_j$	S
(7)	$(S_j \vee S_k)$	S
(8)	$(v_m^n)S_k$	S
(9)	$(\iota v_m^n)S_k$	type n
(10)	$(\lambda n_m^n)S_k$	type $n + 1$

(In the actual writing of formulas, we omit parentheses in accordance with the usual conventions.)

C. Unofficial abbreviations in L_3, introduced by definition schemata.

Conjunction:
(1) $(S_i \,.\, S_j) \equiv \sim(\sim S_i \vee S_j)$.
Conditional:
(2) $(S_i \supset S_j) \equiv (\sim S_i \vee S_j)$.
Existential quantifier:
(3) $(\exists\, v^n_m)S_i \equiv \sim(v^n_m) \sim S_i$.
(4) For $k \geqq 1$, $\{v^n_1, v^n_2, \ldots, v^n_k\} \equiv (\lambda v^n_{k+1})\,[v^n_{k+1} \equiv v^n_1) \vee (v^n_{k+1} \equiv v^n_2) \vee \ldots \vee (v^n_{k+1} \equiv v^n_k)\,]$.
This is the customary notation for finite classes defined by enumeration. (Note that in L_3 only homogeneous classes occur, i.e. those whose elements belong to the same type.)
Recursive definition for iterated unit class formation:
(5) (a) ${}^0(v^n_m) \equiv v^n_m$;
(b) ${}^{p+1}(v^n_m) \equiv \{{}^p(v^n_m)\}$.
Thus, e.g., ${}^3(a^0_1)$ is the class $\{\{\{a^0_1\}\}\}$ of type 3.

This notation can be used for the formation of a homogeneous class expression out of terms of different types. An ordered k- tuple can be defined as a certain class expression of type $n + 3$, where n is the highest type of the members of this tuple. A k-adic relation R can now be construed as a class of ordered k-tuples, thus as a class of type $n + 4$, if n is the highest type of the members of R.

XXIII. Preliminary explanations of models for L_3

A. The use of models. We shall later define 'L-truth' as an explicatum for logical truth, i.e., truth in all possible states (of the universe of discourse). In a simple language (e.g., L_2), even if the domain of individuals is denumerably infinite, it is possible to represent every possible state by a *state-description*, i.e., an infinite class of sentences which contains, for every atomic sentence S_j, either S_j or $\sim S_j$, but not both, and no other sentences. (For the method of defining the L-concepts with the help of state-descriptions, see *M & N*, sect. 2, and, in greater detail, *Foundations*, sections 18 A, D and 20). For a language like L_3, which contains constants and variables of higher levels, it is not possible to represent all possible states by classes of sentences. Therefore the rules must here refer, not to the state-descriptions, but to the possible states themselves. The possible states will be construed here, not as propositions (which would require a non-extensional metalanguage, see *Intr. Sem.*, sect, 18), but as *models*. A *model* for L_3 is a function which assigns to every descriptive constant in L_3 an extension of the corresponding type. [Concerning the use of models, see: Tarski, 1956, ch. VIII (concept of truth) and ch. XVI (logical consequence); Carnap, *Syntax*, ch. III C; Kemeny, "Models" (*J.S.L* **13**, 1948, 16–30), 1953, and 1956.

B. *The individual constants in* L_3 are regarded as logical signs, since numbers are taken as individuals. Therefore the rules will assign to each c^0_m one fixed extension (viz., the number m) in all models. Any identity sentence with two different c^0, e.g. '$a^0_1 \equiv a^0_2$', will then turn out to be logically false.

C. Logical and descriptive definitions of extensions and models.

1. *An extension* may be specified in M either in logical or in descriptive terms. For example, for a number (type 0): '4' is logical, 'the one number (position) which is blue and cold' is descriptive; for a class of numbers (type 1): '$\{3, 5\}$' (i.e., '(λn) ($n = 3$ or $n = 5$)') is logical, '(λn) (n is blue)' is descriptive. '(λn) (n is blue) $= \{3, 5\}$' is a factual sentence; it says that the (descriptively specified) property (λn) (n is blue) has the (logically specified) extension $\{3, 5\}$.
2. *A model* for L_3 may likewise be specified in M either in logical or in descriptive terms. For the definitions of the L-concepts and A-concepts (which will be given in XXIV) the models may be regarded as logical models (sometimes

called "mathematical models"). Likewise, the value assignments for variables (*VA*, in XXIV) may be regarded as logically specified. Only later, in the rules of interpretation and the definition of truth, shall we refer to descriptively defined extensions and models.

D. On A-postulates.

Suppose that, by virtue of the intended meanings of the descriptive constants in L_3, logical dependencies hold between two predicates (e.g., logical implication or incompatibility) or a structural property (e.g., transitivity) holds for a relational predicate. Then some models do not represent possible states. In this case the rules for A-truth (logical truth in the wider sense, analyticity) and other A-concepts must give A-postulates (meaning postulates) for L_3, which express the dependencies and structural properties (XXIV B). The admissible models or A-models will then be defined as those models in which the A-postulates hold. The A-models represent the possible states. Therefore we can then define a sentence as A-true iff it holds in all A-models. *Read*: Carnap, "Meaning postulates".

XXIV. The A-concepts for L_3

A. Value assignments and models.

1. *A value assignment* (*VA*) for the variables in L_3 is a function which assigns to every variable v^n_m in L_3 an extension of type *n*. We use 'VA_r' and 'VA_s' as variables for *VA* in *M*. We write '$VA_r(v^n_m)$' for 'the extension assigned to the variable v^n_m by VA_r'.
2. The value assignment VA_0 is defined as follows (it assigns the same extension to all variables of the same type). For any v^0_m of type 0, $VA_0(v^n_m) = 1$; for any variable v^n_m of any type $n > 0$, $VA_0(v^n_m)$ is the null class of type *n*.
3. A *model* for L_3 is a function which assigns to every con-

stant of every type $n > 0$ an extension of type n. We use in M 'Mod_p' and 'Mod_q' as variables for models. We write '$Mod_p(c^n_m)$' for 'the extension assigned to c^n_m by Mod_p'.

B. The A-postulates of L_3.

1. The following sentences are A-postulates of L_3:

(1) $(v^0_1) \sim (c^1_2(v^0_1) \cdot c^1_3(c^0_1))$.
(b) $c^2_1(c^1_2)$.
(c) $c^2_1(c^1_3)$.
(d) $(v^0_1)(v^0_2)(v^0_3)(c^4_1 \langle v^0_1, v^0_2 \rangle \cdot c^4_1 \langle v^0_2, v^0_3 \rangle \supset c^4_1 \langle v^0_1, v^0_3 \rangle)$.
(e) $(v^0_1 (v^0_2) (c^4_1 \langle v^0_1, v^0_2 \rangle \supset \sim c^4_1 \langle v^0_2, v^0_1 \rangle)$.

2. Explanations. In what follows we shall assume that the whole class of A-postulates of L_3 has been specified, either by enumeration (if the class is finite) or else by schemata or rules in M. The five examples (a) to (e) given above state (a) the incompatibility of c^1_2 and c^1_3; (b) and (c) the membership of the classes designated by c^1_2 and c^1_3, respectively, in the class designated by c^2_1; (d) the transitivity of c^4_1; (e) the asymmetry of c^4_1. [The rules of direct designation to be given later (XXVI A) stipulate that the designata of the constants c^1_1, c^1_2, c^1_3, c^2_1, and c^4_1 are, respectively, the classes Cold, Blue, Red, Color, and the relation Warmer. Therefore the five A-postulates given above are in agreement with the interpretation of the constants stated by the rules of designation, since (a) Blue and Red are incompatible, (b) and (c) Blue and Red are colors, (d) and (e) the relation Warmer is transitive and asymmetric; and this holds, not as a matter of contingent fact, but in virtue of the meanings of the terms. Note, however, that in our present context, i.e., the definition of the A-concepts on the basis of the A-postulates, the rules of designation are not used.]

C. Rules of relative designation for L_3.

For every designator formula D_m in L_3, the following rules (1a) to (10) (for the forms listed in the rules of formation, XXII B) determine $des_{pr}(D_m)$, i.e., the designators of D_m with respect to Mod_p and VA_r. If D_m is of type n, $des_{pr}(D_m)$ is an extension of type n. 'D^n' refers to designator formulas of type n. (For explanations of the rules, see D below.)

	D_m	$des_{pr}(D_m)$
(1a)	c^0_m	m
1b)	c^n_m (for any $n > 0$)	$Mod_p(c^n_m)$
(2)	v^n_m	$VA_r(v^n_m)$
(3)	$D^{n+1}_j(D^n_k)$	$des_{pr}(D^n_k)$ is an element of the class $des_{pr}(D^{n+1}_j)$.
(4)	$D^n_j \equiv D^n_k$	$des_{pr}(D^n_j) = des_{pr}(D^n_k)$.
(5)	$S_j \equiv S_k$	$des_{pr}(S_j) = des_{pr}(S_k)$.
(6)	$\sim S_j$	Not $des_{pr}(S_j)$.
(7)	$S_j \vee S_k$	$des_{pr}(S_j)$ or $des_{pr}(S_k)$.
(8)	$(\lambda v^n_m)S_j$	The class (λu^n) $(des_{ps}(S_j))$, where VA_s is that VA which assigns to v^n_m the extension u^n and otherwise is like VA_r.
(9)	$(\iota v^n_m)S_j$	The one extension u^n (of type n) such that either (a) u^n is the only element of the class $des_{pr}((\lambda v^n_m)S_j)$, or (b) this class does not have exactly one element and $u^n = VA_0(v^n_m)$.
(10)	$(v^n_m)S_j$	Every extension of type n is an element of $des_{pr}((\lambda v^n_m)S_j)$.
(11)	For a class K_m of sentential formulas, $des_{pr}(K_m) =_{\text{Df}}$ for every element S_j of K_m, $des_{pr}(S_j)$.	
(12)	For a designator D_m, $des_p(D_m) =_{\text{D}f} des_{p0}(D_m)$ (referring to VA_0, see A2 above.).	
(13)	For a class K_m of sentences, $des_p(K_m) =_{\text{D}f} des_{p0}(D_m)$.	

D. Remarks on the rules in C.

1. For any D_m in L_3, there is one of the rules C(1a) to (10) which yields a sentence in M of the form '$des_{pr}(D_m) = \ldots$',

where a designator formula A_i of M stands in the place of the three dots; if D_m is a designator formula of type n in L_3, A_i is a formula of type n in M; and if D_m is a sentential formula in L_3, A_i is a sentential formula in M. [For example, for the individual constant c_3^0, we obtain by the rule C(1a): '$des_{pr}(c_3^0) = 3$'; for $v_3^0 \equiv v_5^0$, we obtain from C(4): '$des_{pr}(v_3^0 \equiv v_5^0 = (des_{pr}(v_3^0) = des_{pr}(v_5^0))$'.] If the formula A_i contains 'des_{pr}', A_i is transformed by further rules (in the second example, C(2) is applied twice), until finally a formula A_i is obtained which does no longer contain 'des_{pr}'. If a particular model, say Mod_3, and a particular VA, say VA_5, are defined in M in logical terms (i.e., without the use of descriptive terms of the translation vocabulary, like 'blue'), then for any D_m we obtain finally a result of the form '$des_{3,5}(D_m) = \ldots$', where a logical designator in M stands in the place of the three dots.

2. Note that '$des_{pr}(S_j)$' is a sentential formula in M. We have (see XX Gl (c))
   (a) '$des_{pr}(S_j)$' and '$des_{pr}(S_j) = T$' are logically equivalent in M.
   (b) 'not $des_{pr}(S_j)$' and '$des_{pr}(S_j) =$ non-T' are logically equivalent in M.

   Therefore an expression in M of the form '$des_{pr}(S_j) = \ldots$', where a sentential formula in M stands in the place of the three dots, may be transformed in '$des_{pr}(S_j) = T) = \ldots$'; thus it may be read, not only as 'the designatum of S_j with respect to Mod_p and VA_r is ...', but also as 'S_j is true with respect to Mod_p and VA_r, iff ...' or as 'S_j is satisfied by VA_r and Mod_p iff ...'.

3. Remarks on particular rules.
   (a) The rules C(2), (6), (7), and (10) (the latter in combination with (8)) are analogous to some rules for L_2 (viz., XX D, RD 3, 5, 6, 7 respectively). C(3) is a generalized analogue to RD4.
   (b) C(9) is in agreement with the convention on iota-descriptions stated in XXI D (for VA_o, see A2).

(c) The rules C(11) and (13) are based on the conjunctive interpretation of a class of sentences; a class is regarded as true iff every sentence belonging to it is true (*Intr. Sem.*, p. 34).

(d) It is easily seen from the rules that, if D_m is a designator and thus without free variables, then $des_{pr}(D_m)$ depends only on Mod_p but is independent of VA_r. Therefore we may use here a function des_p instead of des_{pr}. In its definition an arbitrarily chosen VA may be used; in C(12) we have taken VA_0.

E. L-concepts and A-concepts.

The term 'L-truth' is here used for logical truth in the narrower sense, i.e., truth based on the meanings of the logical constants (e.g., a sentence of the form $S_1 \vee \sim S_1$ is L-true). I use the term 'A-truth' for logical truth in the wider sense, or analytic truth, i.e., truth based on the meanings of the logical constants and on the relations between the meanings of descriptive constants expressed by A-postulates (e.g., a sentence of L_3 saying that, if the position 3 is blue, it is not red, is A-true but not L-true). Since L_3 contains A-postulates, the A-concepts are here more important than the L-concepts. We shall give only definitions of A-concepts. The definition of an L-concept is analogous to that of the corresponding A-concept; it is obtained from the later by replacing the prefix 'A–' throughout by 'L-', and 'A-model' by 'model'. Thus the L-concepts are based on the totality of all models, while the A-concepts are based on the narrower class of A-models, i.e., those models which satisfy all A-postulates.

1. Mod_p is an *A-model* (admissible model) for $L_3 =_{\mathrm{Df}} des_p(K_A)$, where K_A is the class of the A-postulates in L_3.
2. A designator D_m is an *A-determinate designator* $=_{\mathrm{Df}}$ for any two A-models Mod_p and Mod_q, $des_p(D_m) = des_q(D_m)$.
3. c^n_m is a *logical designator constant* $=_{\mathrm{Df}}$ c^n_m is A-determinate. Otherwise c^n_m is called a non-logical or *descriptive* designator constant.

4. D_m is a *descriptive designator formula* $=_{\text{Df}}$ D_m contains at least one descriptive designator constant. Otherwise, D_m is a *logical* designator formula.

The following definitions (5) through (8) refer to *sentential formulas.*

5. S_j is *A-valid* (in L_3) $=_{\text{Df}}$ for every A-model Mod_p and every VA_r, $des_{pr}(S_j)$.
6. S_j is *A-contravalid* $=_{\text{Df}} \sim S_j$ is A-valid.
7. S_j *A-implies* $S_k =_{\text{Df}} \sim S_j \vee S_k$ is A-valid.
8. S_j is *A-equivalent* to $S_k =_{\text{Df}} S_j \equiv S_k$ is A-valid.

The following definitions refer to sentences.

9. A sentence S_j is *A-true* $=_{\text{Df}}$ S_j is A-valid.
10. A sentence S_j is *A-false* $=_{\text{Df}} \sim S_j$ is A-true.
11. A sentence S_j is *A-indeterminate* $=_{\text{Df}}$ S_j is neither A-true nor A-false.

F. Theorems.

On the basis of the rules and definitions stated, theorems for A-concepts hold in analogy to the usual theorems for L-concepts (e.g., in *Intr. Sem.*, sect. 14, the postulates P14–5 through 9, and 11 through 15.)

We have furthermore:

1. A sentence S_i is A-determinate iff S_i is either A-true or A-false.
2. If the designator D_m contains no descriptive designator constant, D_m is A-determinate (and, moreover, L-determinate).

The definition E3 is adequate also for systems in which the models include the individual constants. Since this is not the case for L_3 (see A3), here the theorems (3) and (4) hold:

3. Any c_m^0 in L_3 is a logical designator constant. (From C(1a).)
4. If the designator D_m in L_3 contains no designator constant of any type $n > 0$, D_m is A-determinate.
5. A sentence S_j is A-true iff, for every A-model Mod_p, $des_p(S_j)$.

G. Example.

Let S_1 be $c_1^1(v_1^0)$, S_2 be $S_1 \vee \sim S_1$, and S_3 be $(v_1^0)(S_2)$. By C(6) and (7), the following holds for any Mod_p, and VA_r, and any VA_s:

(a) $des_{pr}(S_2) = des_{pr}(S_i)$ or not $des_{pr}(S_1)$.

(b) $\qquad = T$.

(c) $des_{ps}(S_2) = T$.

From C(8):

(d) $des_{pr}((\lambda v_1^0)S_2) =$ the class of all numbers u^0 such that $des_{ps}(S_2)$, where VA_s is that VA which assigns to v_1^0 the number u^0 and otherwise is like VA_r.

Hence with (c):

(e) $des_{pr}((\lambda v_1^0)S_2) =$ the class of all numbers u^0 such that T,

(f) $\qquad =$ the class of all numbers.

Hence with C(10):

(g) $des_{pr}(S_3) =$ every number is an element of the class of all numbers,

(h) $\qquad = T$.

From (b) and (h) by D2 (a):

(i) $des_{pr}(S_2)$,

(j) $des_{pr}(S_3)$.

From these with E 5:

(k) S_2 is A-valid (and, moreover, L-valid).

(l) S_3 is A-valid.

Hence with E 9:

(m) S_3 is A-true (and, moreover, L-true).

XXV. Preliminary remarks on interpretation and truth for L_3

A. On the rules of interpretation. A complete statement of these rules (which is not intended here) would have to include the following specifications:

(1) a specification of the domain of individuals, here positions,

(2) a specification of the enumeration of the positions, i.e., for every n, an explanation as to which position is taken as the position No. n,

(3) a specification of the meanings of the predicate constants $c^n (n > 0)$.

We shall omit here the points (1) and (2), assuming that they have been specified. We shall give (in XXVI A) a few examples of rules of the kind (3), merely to illustrate the form of these rules.

We assume that L_3 contains only a finite number of predicate constants. For each of them its *direct designatum* is to be stated by a rule of the form '$ddes(c^n_m) = \ldots$', where '...' is a predicator in M of the type n. In order to fulfill the requirement of adequacy (see D below), this predicator '...' in M must be a translation of c^n_m according to the intended interpretation. The designata stated in the rules must furthermore be in agreement with the class of A-postulates for L_3 (of which we have given some examples in XXIV B1); more specifically, all logical relations and properties which hold for the designata, and no others, must be expressed by the A-postulates. According to the rule XXVI A3(a) (below), c^4_1 designates the relation Warmer, which is logically transitive; thus this rule is in agreement with the A-postulate XXIV B1(d). We assume that all the designata assigned by the *ddes*-rules for L_3 are descriptive; this is the case for the rules stated in XXVI A.

B. 1. The *ddes*-rules constitute a definition of the function *ddes* for L_3. This function assigns to every primitive constant c^n in L_3 (for $n > 0$) a class of type n as its extension, specified in descriptive terms. Thus *ddes* is a model for L_3 (XXIV A3). However, it is not a logical model, like those previously considered, but a *descriptive model* (i.e., the constant '*ddes*' in M is not logical, but descriptive). Since we assume that the *ddes*-rules are in agreement with the A-postulates, *ddes* is an A-model.

2. There is exactly one logical model Mod_p such that $Mod_p = ddes$; that is to say, such that, for any primitive constant $c^n_m (n > o)$, the logically specified class assigned by Mod_p to c^n_m happens to be identical with the descriptively specified class $ddes(c^n_m)$. The identity between this logical

model and *ddes* can only be determined on the basis of factual knowledge (see the example XXVI CI below).

C. On the basis of *ddes*, the general designation function *des*, applicable to all designators, will be defined (XXVI B1). The function *des* is the special case of des_p (XXIV C(12)) for that Mod_p which is identical with *ddes*. Thus the function *des* assigns to any designator that designatum which is based on the direct designata assigned to the primitive predicate constants by *ddes*.

D. If designation is expressed by a functor, the uniquences condition is necessarily fulfilled (because, from '$des(D_m) = \ldots$' and '$des(D_m) = - . -$', '$\ldots = - . -$' follows). Therefore, the following *sufficient and necessary condition of adequacy for designation in L* takes now the place of the earlier one (XV C), again for all three metalanguages $M^{\circ\circ}$:
A functor '*d*' in $M^{\circ\circ}$ is an adequate functor for designation$^{\circ\circ}$ in L iff, for every designator D_m in L, a sentence in $M^{\circ\circ}$ of the form '$d(\ldots) = ---$', with a spelling description of D_m in the place of '...' and a translation of D_m into $M^{\circ\circ}$ in the place of '$---$', follows from the definition or the rules for '*d*'.

E. A sentence will be said to be *true* iff its designatum holds (XXVI B3). This definition is essentially the same as XI B(1). [The earlier definiens would be reformulated with the functor '*des*' for sentences as 'there is a p such that $p = des(S_j)$ and p'; and this is logically equivalent in M to '$des(S_j)$'.]

XXVI. Rules of interpretation and truth for L_3

A. 1. Rules of direct designation (*ddes*) for L_3.
(a) $ddes(c_1^1) = (\lambda x)(x \text{ is cold})$,
(b) $ddes(c_2^1) = (\lambda x)(x \text{ is blue})$,
(c) $ddes(c_3^1) = (\lambda x)(x \text{ is red})$,
etc.
2. For predicates of type 2:

(a) $ddes(c_1^2) = (\lambda x_1^1)$ (x_1^1 is a color),
etc.

3. For predicates of type 4:

(a) $ddes(c_1^4) = (\lambda x_1^3)\ (\lambda x_1^0)\ (\exists x_2^0)\ (x_1^3 = \langle x_1^0, x_2^0 \rangle$ and x_1^0 is warmer than x_2^0)),
etc.

Similar rules may be stated for constants of other types.

B. Definitions of designation and truth for L_3.

1. For any designator D_m, $des(D_m)$ in $L_3 = des_p(D_m)$ for $Mod_p = ddes$.
2. Theorems.

(a) For any primitive descriptive constant $c_m^n (n > o)$, $des(c_m^n) = ddes(c_m^n)$. (From (1) and XXIV C(1b).)

(b) For any c_m^0, $des(c_m^0) = m$. (From XXIV C(1a).)

3. For any sentence S_j, S_j is true in $L_3 =_{Df} des(S_j)$.

C. Examples. We assume for these examples that L_3 contains only two primitive descriptive constants, viz., c_1^1 and c_2^1, with the *ddes*-rules A 1(a) and (b). (In this case, there are no A-postulates.)

1. Example for *ddes* and models. Let us suppose that, as a matter of *fact*, the positions 1, 2, and 3 are the only cold ones and that the positions 3 and 5 are the only blue ones. Thus the following two class identities hold factually (see XXIII c1):

(a) (λx) (x is cold) $= \{1, 2, 3\}$,

(b) (λx) (x is blue) $= \{3, 5\}$.

From these we derive with the rules A for *ddes* two factual sentences:

(c) $ddes(c_1^1) = \{1, 2, 3\}$,

(d) $ddes(c_2^1 = \{3, 5\}$.

These sentences give the actual extensions of the two constants. Now there is in M a logical model constant, say

'Mod_8', which is defined by the following sentences (e) and (f):

(e) $Mod_8(c_1^1 = \{1, 2, 3\}$,

(f) $Mod_8(c_2^1) = \{3, 5\}$.

Thus the constant c_1^1 has in Mod_8 the same extension as in the model *ddes*; and c_2^1 has also in both models the same extension. Hence the descriptive model *ddes* is factually identical with the logical model Mod_8:

(g) $ddes = Mod_8$.

We may say that *the true model* (or the actual model), i.e., the model which ascribes to the primitive constants those extensions which they (in their intended interpretations) actually have, is descriptiviely specified in *ddes*, and logically specified in Mod_8.

Note that the interpretation of the constants is given only by the *ddes*-rules in A above, but not by the factual sentences (c) and (d) without those rules, let alone by the definition of the logical model constant 'Mod_8' (in (e) and (f)).

2. Example for *des* and truth. Let S_5 be the sentence $c_2^1(c_3^0)$ in L_3.

From A 1(b) and B2(a) and (b):

(a) $des(c_2^1 = (\lambda x)$ (x is blue),

(b) $des(c_3^0) = 3$.

Hence with XXIV C(e):

(c) $des(S_5) =$ (3 is an element of (λx) (*x* is blue)).

From the definition of truth (B3):

(d) S_5 is true $= des(S_5)$,

(e) S_5 is true iff 3 is an element of (λx) (*x* is blue).

The right-hand side of (c) and of (e) is a translation of S_5; thus the conditions of adequacy both for designation (XXV D) and for truth (XVI B) are here fulfilled.

Now we take (b) in Ex. 1 as *a factual premise.* This yields:

(f) 3 is an element of $(yx))$ (*x* is blue).

Hence with (c):

(g) S_5 is true.

XXVII. The controversey on meaning and analyticity

Semantics may be divided into two parts:

(1) *the theory of extension*, dealing with concepts like extension (designation^e, XIII), denotation (in the sense of 'Des^e', XVII), satisfaction, truth, naming, and related ones.

(2) *the theory of intension* (or meaning) dealing with concepts like meaning (or intension or sense as possible explicata, and the relations Des^i and Des^s), logical truth and analyticity (L-truth as explicatum), synonymy, and related ones.

Some philosophers, while accepting (1), reject all concepts of the kind (2). See Quine (1953), esp. chs. II, III, VII and White (1960).

Replies defending the meaning concepts: Mates (1951), Martin (1952). (Quine (1953), pp. 35 and 138 makes brief comments on Martin.)

It seems advisable to distinguish two problems:

(a) the problem of meaning concepts *for artificial language systems* defined by their rules,

(b) the problem of meaning concepts *for natural languages.*

The first problem and especially that of explicating logical truth in the wider sense (analyticity) can be solved by special semantical rules, e.g., *meaning postulates.* (See XXIII D, XXIV B and Carnap (1952).

Quine (1953, ch. III) admits the possibility of laying down special semantical rules for meaning concepts. But he doubts whether they explicate meaning, unless there are, as explicanda, meaning concepts which can be applied to *natural languages* on the basis of behavioristic criteria like other concepts of empirical linguistics. I have discussed a concept of this kind, viz., the pragmatical concept of intension, in Carnap (1955).

XXVIII. Intensions and quasi-intensions

A. Intensions in M^i.

A few brief remarks about the intensional metalanguage M^i (XIII) will be made in this section. In M^i, in distinction to

M^e, statements about intensions can be formulated, e.g., a statement saying that the intension (designatum^i) of a given designator is such and such.

Since the variables in M^i have intensions as values, there are also general statements such as "For every proposition, . . ." or "There is a property of the type $[t_1]$ such that . . ." and the like. For each type, these general statements refer to a range of intensions which is much more comprehensive than the class of these intensions for which there are designators in the object language L (say, L_3).

In M^i, a system of *modal logic* is used. We may take 'N' as the primitive modal sign, a logical constant. "N(. . .)" stands for "it is logically necessary that . . ." (M & N, ch. V).

B. Quasi-intensions in M^e.

The sentences of M^e refer directly to extensions only, not to intensions. But there is a one-one correspondence between the intensions and a special kind of extensions, which we shall call quasi-intensions, such that the logical and semantical properties of any intension are mirrored by those of the corresponding quasi-intension. The following definitions refer to L_3: for other language forms, analogous definitions can be formulated.

(1) The *quasi-intensions* corresponding to the type t (for L_3) $=_{\mathrm{Df}}$ the functions from admissible models (for L_3) to extensions of the type t (XXI B).

Note that the quasi-intensions corresponding to the type t are themselves extensions, not of the type t, but of another type of higher level.

(2) *The quasi-intension of the designator* D_k (in L_3) $=_{\mathrm{Df}}$ that function which, for any admissible model Mod_p, has the value $des_p(D_k)$. (For des_p, see XXIV C(12).)

[It is here assumed, for the sake of simplicity, that in M^i 'N' does not occur in the operand of a λ-operator or an ι-operator. If such occurrences are admitted, the ranges of intensions in M^i and the corresponding ranges of quasi-intensions in M^e must be still more comprehensive, and the definitions (1) and

(2) must be replaced by others which are somewhat more complicated. The guiding idea for the translation indicated in C remains, however, the same.]

C. Translation from a modal language L^i into an extensional language L^e.
If a language L^i with logical modalities and intension variables (similar to M^i) is given, it is possible to specify an effective method by which any sentence of L^i is translated into an L-equivalent sentence of an extensional language L^e. We shall not specify the method here but give only rough indications of the translation for the two most important forms of sentential formulae. A universal formula of the form "for every intension of the type *t*, . . ." in L^i is translated into one of the form "for every quasi-intension corresponding to the type *t*, . . ." in L^e. And a modal formula "*N*(. . .)" is translated into one of the form "for every admissible model Mod_p, . . .". This translation is plausible since a proposition is logically necessary iff it holds in every possible case, that is, in every admissible model (*M & N*, p. 186).
Even for those who accept only an extensional language, logical modalities and intension variables are shown by this translation to be unobjectionable, provided that the variables in the extensional language L^e have sufficient ranges of extension values to accommodate the quasi-intensions corresponding to the intensions which are values of variables in L^i. In particular, the method indicated can be used for translating any sentence of M^i into one of M^e. This legitimizes the semantics of intensions for an extensionalist point of view.

XXIX. The controversy about abstract entities in semantics

A. *A nominalistic language* is one in which all values of all variables are concrete (say, observable objects or events). There is a controversy among analytic philosophers today about the legitimacy of non-nominalistic languages. Some, e.g. Tarski, Quine and Goodman, deny or doubt that a language which is non-nominalistic and not translatable into a nominalistic one can be accepted as meaningful. (See Quine (1953), esp. ch. VI.) In this contemporary controversy sometimes the old terms "nominalism" and "universalism" (or "realism" or "Platonism") are used. This seems to me inadvisable. The earlier controveries were formulated in a very unclear way, and it seems even doubtful whether the philosophical ("ontological") assertions which were discussed under these labels had any cognitive content.

Since sentences about intensions are translatable into sentences about extensions (XXVIII), the controversy concerns essentially the admissibility of variables for abstract (i.e., non-concrete) extensions of various kinds, e.g., classes (of objects), classes of classes (of objects), relations, numbers, functions, etc.

B. In my view, the introduction of variables of a new kind is a matter of practical decision. Certain theoretical investigations are certainly relevant for the decision, e.g., investigations of the logical and semantical features, both the desirable and the undesirable ones, of the enlarged language. Among the features to be considered may be, e.g., the simplicity of the logical structure, the strength in means of expression and means of deduction, the danger of inconsistency, and the like. (There is never any absolute certainty of consistency; and the degree of confidence in consistency often decreases by the introduction of a new kind of abstract variables.) But the legitimacy of the introduction is not dependent upon an alleged prior metaphysical insight into the "existence" or "ontological reality" of the new entities.
Read: Carnap (1950).

C. To the arguments in the paper mentioned I would today add the suggestion that it might be advisable to regard the *meta-language M* for syntax and semantics *as part of the theoretical language*, not of the observation language. (I would prefer to do this even if the object language is part of the observation language.) A given language community may well decide to admit in their common observation language L_0 only sentences which are completely understood by all members of the community and, therefore, to lay down some or all of the following *requirements for* L_O*:*

(1) Requirement of observability. All primitive descriptive predicates in M_O designate properties or relations which are directly observable for all members of the community.
(2) Nominalistic requirement. The values of the variables are observable objects (or object–moments or events).
(3) Wide requirement of finitism. The rules of L_O do not state or imply that the domain of individuals is infinite.

On the other hand, for the *theoretical language* L_T we can never have more than an incomplete interpretation. There is no reason to restrict this language by requirements similar to those for L_O. On the contrary, we should admit in L_T all means of expression and of deduction which are found to be useful for the purpose of this language, which is, to supply a theoretical superstructure for L_O.
This applies also to the semantical *metalanguage* M, now regarded as part of L_T. For example, even if the object language is the nominalistic observation language, we should feel free to admit in M variables for classes of objects (and, if it seems useful, also variables for classes of classes, for functions, for intensions or quasi-intensions) and to use these variables in the definitions of semantical concepts, e.g., the concepts of designation, extension, intension, truth, model, L-truth, etc.

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